Math 4329, Final

Name ______________________________

1. a. Let $T_3(x)$ be the Taylor polynomial of degree 3 which matches $f(x), f'(x), f''(x)$ and $f'''(x)$ at $x = 0$, where $f(x) = cos(x/3)$. Find the best possible bound on

   $max_{-0.5 \leq x \leq 0.5} |T_3(x) - f(x)| \leq$

   b. Let $L_3(x)$ be the Lagrange polynomial of degree 3 which matches $f(x)$ at $x = -2, -1, 1$ and 2, where $f(x) = cos(x/3)$. Find the best possible bound on

   $|L_3(0) - f(0)| \leq$

2. a. A root finder gives consecutive errors of $e_8 = 10^{-3}, e_9 = 10^{-5}, e_{10} = 10^{-11}$. Estimate the order of the method.

   b. A quadrature method gives an error of 10^{-5} when $h = 10^{-2}$ and 10^{-11} when $h = 10^{-4}$. Estimate the order of the method.

3. Use the inverse power method to determine the smallest eigenvalue of

$$A = \begin{bmatrix} \frac{4}{6} & \frac{-2}{6} \\ \\ \frac{-5}{6} & \frac{1}{6} \end{bmatrix}$$

Start the iteration with $(x_0, y_0) = (3, 8)$.

4. Take one step of a third order Taylor series method (Euler is the first order Taylor method) to approximate the solution of the following problem, at $t = 1.01$:

$u' = 2tu$
$u(1) = 1$

5. Do one iteration of Newton's method, starting from $(0, 0)$, to solve:

$f(x, y) = 2x^2 + y - 1 = 0$
$g(x, y) = -2x + y^2 + 1 = 0$

6. Write $\sqrt{1+x} - 1$ in a form where there is no serious problem with roundoff, when $x \approx 0$.

7. Determine the degree of precision of the quadrature rule:
$\int_0^h f(x)dx \approx \frac{h}{4}f(0) + \frac{3h}{4}f(\frac{2h}{3})$

8. Will the iteration $x_{n+1} = 2 - \frac{3}{2}x_n + \frac{1}{2}x_n^3$ converge to the root 1, if the starting guess is sufficiently good? **Justify** your answer.