Math 4329, Final

Name ____________________________

1. Use the power method to find the largest (in absolute value) eigenvalue of

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 10 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Start with $(1, 5, 1)$ and do 3 iterations. What is the corresponding eigenvector?

2. a. Write the third order differential equation $u''' + 4u'' + 5u' + 2u = 2t^2 + 10t + 8$ as a system of three first order equations, that is, in the form:

$u' = f(t, u, v, w) =$

$v' = g(t, u, v, w) =$

$w' = h(t, u, v, w) =$

b. Now write out the formulas for $u_{n+1}, v_{n+1}, w_{n+1}$ for Euler's method applied to this system of first order equations:

$u_{n+1} =$

$v_{n+1} =$

$w_{n+1} =$

3. If the third order Taylor series method (two more terms than Euler's method) is used to solve $u' = -u^2$, write u_{n+1} in terms of h, t_n and u_n only. ($t_n = nh, u_n \approx u(t_n)$)

4. Do **one** iteration of Newton's method, starting from $(1, 1)$, to solve:
   $f(x, y) = x^2 + xy^3 - 9 = 0$
   $g(x, y) = 3x^2y - y^3 - 4 = 0$

5. a. A rootfinder produces consecutive root estimates of $2.1, 2.001, 2.000001$, when the exact root is $r = 2$. Estimate the order of the method.

   b. A differential equation approximation produces the approximation $u(1) = 2.001$ when $h = 0.01$ and $u(1) = 2.000001$ when $h = 0.001$. If the true solution is $u(1) = 2$, estimate the order of the method used.

6. How should A, r be chosen to make the approximation:

   $\int_{-1}^{1} f(x)dx \approx Af(-r) + Af(0) + Af(r)$

   as high degree of precision as possible?

7. a. If $p_3(x)$ is the third degree (Lagrange) polynomial which satisfies $p_3(x_i) = f(x_i),\ i = 0, 1, 2, 3$, give a formula for the error $f(x) - p_3(x)$ at an arbitrary point x.

   b. If $T_3(x)$ is the third degree (Taylor) polynomial which satisfies $T_3(x_0) = f(x_0), T_3'(x_0) = f'(x_0), T_3''(x_0) = f''(x_0), T_3'''(x_0) = f'''(x_0)$, give a formula for the error $f(x) - T_3(x)$ at an arbitrary point x.

8. Will the iteration $x_{n+1} = \frac{1}{x_n - 1}$ converge to the root 1.618, if the starting guess is sufficiently good? **Justify** your answer.

9. Consider the linear system:

$$\begin{bmatrix} 1+\epsilon & 1 \\ 1 & 1+\epsilon \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

True or False:

a. If $\epsilon > 0$ the Jacobi iterative method for solving this system will converge to the true solution, no matter what starting point (x_0, y_0) is used.

b. If ϵ is close to 0, this matrix will have a large condition number.