Math 5329, Test I

Name ______________________________

1. a. Find $T_n(x)$, the Taylor series of degree n for the function $f(x) = ln(1+x)$, expanded around $c = 0$.
(Hint: $f^{(n)}(x) = (-1)^{n-1}(n-1)!/(1+x)^n$, for $n \geq 1$.)

b. Find $E_n(x)$, the error in $T_n(x)$, and find a reasonable upper bound on $E_n(1)$.

c. Estimate the number of terms n required for $T_n(x)$ to approximate $f(1) = ln(2)$ to 5 decimal places accuracy.

d. Would you expect roundoff error to be a serious concern in (c)? Why or why not?
(Hint: $1 + 1/2 + 1/3 + 1/4 + 1/5 + ... + 1/n \approx ln(n)$, for large n.)

2. Estimate the order of convergence of a root-finder that has consecutive errors $0.2, 0.08, 0.00512$.

3. If Newton's method is used to find a root of $f(x) = x^2 - R$, find bounds on x_0 for which convergence to the root $\sqrt{R}$ is guaranteed.
(Hint: for Newton's method, $e_{n+1} = \frac{1}{2}[f''(\psi_n)/f'(x_n)]e_n^2$, where ψ_n is between x_n and the root.)

4. The golden search method tries to minimize $f(x)$, where f is assumed to be unimodal in $a \leq x \leq b$, by evaluating f at two points between a and b, $p = a + (1 - r) * (b - a)$ and $q = a + r * (b - a)$, where $r = 0.618....$ If $f(q)$ is larger than $f(p)$, the minimum is known to be in the new interval $[a, q]$, otherwise the minimum is known to be in $[p, b]$. Why? Would this algorithm still work if we used $r = 0.75$? What is the advantage of using $r = 0.618...$?

5. Write out the equations used to solve the following system using Newton's method:
$f(x, y) = 1 + x^2 - y^2 + e^x cos(y) = 0$
$g(x, y) = 2xy + e^x sin(y) = 0$

6. To solve $x^2 - 3x - 4 = 0$ we could write $x^2 = 3x + 4$, then $x = 3 + 4/x$, and iterate with this last formula: $x_{n+1} = 3+4/x_n$. Determine (without actually iterating) if this iteration will converge if we start near the root $r = 4$. What if we start near the root $r = -1$?