Math 5329, Test I

Name ____________________________

1. a. Write the Taylor polynomial $T_n(x)$ of degree n for the function $f(x) = cos(x)$, expanded around $a = 0$.

   b. Find a reasonable upper bound on the error in $T_n(x)$ at $x = 25$ and estimate how big n needs to be for the error to be less than 10^{-3}.

   c. Do you expect to have significant problems with roundoff error in calculating $T_n(25)$, with n as in part b? What if you calculate $T_n(1)$ with the same n?

2. Show that the iteration $x_{n+1} = x_n - \frac{f(x_n)}{q(x_n)}$ converges quadratically (at least) to the root r of $f(x) = 0$, if $lim_{x \to r} q(x) = f'(r) \neq 0$.

3. For a certain root finder (Muller's method) it can be shown that $lim_{n\to\infty}\frac{e_{n+1}}{e_n e_{n-1} e_{n-2}} = M(\neq 0, \neq \infty)$. To estimate the order α of this method, assume $e_{n+1} = Ce_n^{\alpha}$, and $e_{n+1} = Me_n e_{n-1} e_{n-2}$. Find an equation satisfied by α, you need not actually find α.

4. To minimize the function $f(x, y) = 100(x^2-y)^2+(1-x)^2$, set f_x and f_y equal to zero, and do one iteration of Newton's method, starting from $(1, 0)$ to solve this system of two equations and two unknowns. From $(1, 0)$, what is the direction of steepest descent?

5. Explain how Newton's method could be used to compute A/B on a computer which only can add, subtract and multiply, but not divide.

6. If $f(x) = (x - r)^m$, show that the "modified" Newton's method $x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}$ will converge in a single iteration to the root r, regardless of the starting value x_0. What would you predict would happen if this modified Newton method were applied to a more general function with a root of multiplicity m at r, that is to $f(x) = (x - r)^m h(x)$, where $h(r) \neq 0$? You can analyze the iteration using the techniques of section 3.4, or you can guess; but if you guess, it must be correct!