Math 5329, Test I

Name ______________________________

1. a. Find $T_n(x)$, the Taylor series of degree n for the function $f(x) = ln(1+x)$, expanded around $a = 0$.
(Hint: $f^{(n)}(x) = (-1)^{n-1}(n-1)!/(1+x)^n$, for $n \geq 1$.)

b. Find $E_n(x)$, the error in $T_n(x)$, and find a reasonable upper bound on $|E_n(1)|$.

c. Estimate the number of terms n required for $T_n(x)$ to approximate $f(1) = ln(2)$ to 5 decimal places accuracy.

d. Would you expect roundoff error to be a serious concern in (c)? Why or why not?
(Hint: $1 + 1/2 + 1/3 + 1/4 + 1/5 + ... + 1/n \approx ln(n)$, for large n.)

2. Estimate the order of convergence of a root-finder that has consecutive errors $0.2, 0.08, 0.002048$.

3. A certain computer stores floating point numbers in a 128-bit word. If a floating point number is written in normalized binary form $(1.xxxxx..._2 * 2^e)$, it is stored using one sign bit (0 if the number is positive), then $e + 4095$ is stored in binary in the next 13 bits, and then the mantissa xxxxx... is stored in the final 114 bits. Show exactly how the number -27.125 would be stored on this computer. Also: approximately how many **decimal** digits of accuracy does this machine have?

4. The fixed point iteration $x_{n+1} = x_n + sin(x_n)$ has roots at $r = n\pi$ for any integer n. Will this iteration converge if you start very close to the root $r = 0$? Will it converge if you start near the root $r = \pi$? In both cases, if it does converge, what is the order of convergence?

5. Write the secant iteration for solving $f(x) = 1/x - b = 0$, in a form where no divisions are required (thus this iteration could be used to compute $1/b$ on a computer which cannot do divisions).

6. To minimize the function $f(x, y) = 10(2x + y)^2 + (x - 2)^2$, set f_x and f_y equal to zero, and do one iteration of Newton's method, starting from $(0, 1)$ to solve this system of two equations and two unknowns. The true minimum is obvious from looking at the function, where is the minimum? From $(0, 1)$, what is the direction of steepest descent? Which converges faster, Newton's method or the method of steepest descent?