Math 5329, Test II

Name ______________________________

1. a. Find the LU decomposition (without pivoting) for

$$A = \begin{bmatrix} 4 & -8 \\ -8 & 25 \end{bmatrix}$$

b. Now find the Cholesky decomposition LL^T of the positive definite matrix A. (Hint: First, write LU in the form LDU_2), where D is the diagonal of U, and $U_2 = D^{-1}U$ and note that $U_2 = L^T$.)

2. Find the condition number (using the L_∞ norm) of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ & \\ 2 & 4 + 10^{-10} \end{bmatrix}$$

If machine precision is about 10^{-16} about how many significant digits should we expect when $Ax = b$ is solved, using Gaussian elimination with partial pivoting?

3. Do several iterations of the inverse power method to find the smallest eigenvalue (in absolute value) of A, and the corresponding eigenvector, if

$$A = \begin{bmatrix} 1 & \frac{1}{2} \\ \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}$$

4. If $p_2(x)$ is the polynomial of degree 2 which interpolates $f(x) = ln(x)$ at $x = 0.5, 1.0, 1.5$, find reasonable upper *and* lower bounds on $|p_2(2) - ln(2)|$.

5. Determine a,b,c,d,e so that

$$
s(x) = \begin{cases} a(x-2)^2 + b(x-1)^3 & x \leq 1 \\ c(x-2)^2 & 1 \leq x \leq 3 \\ d(x-2)^2 + e(x-3)^3 & 3 \leq x \end{cases}
$$

is the cubic spline interpolant through the points (0,2),(1,4),(3,4) and (4,2). (There is a unique solution, even though no end conditions are specified.)