Math 5329, Test II

Name _____

1. a. Find the LU decomposition (no pivoting necessary) for

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}$$

b. What is an LU decomposition good for?

c. Find the Cholesky decomposition LL^T of A.

d. Show that a matrix that has a Cholesky decomposition $A = LL^T$, where L nonsingular, is positive definite.

2. If $A = L + D + U$ (L = strictly lower triangular, U is strictly upper triangular and D is diagonal), what is the matrix whose eigenvalues must be less than one in absolute value for convergence, for the

   a. Jacobi Iterative Method?

   b. Gauss-Seidel Iterative Method?

3. To find the eigenvalue of A closest to a number p, the power method can be applied to what matrix? Explain how an LU decomposition could be used to make this iteration more efficient.

4. If $p_N(x)$ is the polynomial of degree N which interpolates $f(x) = e^{3x}$ at $N + 1$ uniformly spaced points between 0 and 10, find a bound, involving only N, on $max(0 \leq x \leq 10)|p_N(x) - f(x)|$ Will your bound go to zero as $N \to \infty$?

5. What is the condition number (using the L_∞ norm) of

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 + 10^{-9} \end{bmatrix}$$

If our computer has about 20 decimal digits precision, about how many significant decimal digits would we expect in the solution of $Ax = b$?

6. A quintic spline interpolant is a function which is a polynomial of degree five or less in each interval $(x_{i-1}, x_i), i = 1, ..., N$ and passes through the points $(x_i, y_i), i = 0, ..., N$ and is continuous and has continuous first, second, third and fourth derivatives.

   a. How many unknown coefficients need to be determined? (Hint: There are N intervals and the quintic has how many coefficients in each?)

   b. How many interpolation conditions are there? (Hint: There are two interpolation conditions for each interval.)

   c. How many continuity conditions are there? (Hint: s(x) is automatically continuous because of the interpolation conditions, so we only need to require that s', s'', s''', s^{iv} be continuous at each *interior* point—how many interior points are there?)

   d. If you add the number of interpolation conditions (part b) and continuity conditions (part c), does this equal the number of unknowns (part a)? If not, what needs to be done to make the quintic spline interpolation problem have a unique solution?