Math 5329, Test III

Name ______________________________

1. a. Find r, s which make the quadrature formula below as high order as possible:

   $\int_a^b f(x)dx \approx \sum_{i=1}^{N} \frac{h}{3}[f(x_{i-1}+rh) + f(x_{i-1}+\frac{1}{2}h) + f(x_{i-1}+sh)]$

   (Hint: how are r and s related, by symmetry?)

   b. With this choice for r, s, what is the global order of this rule?

2. a. Is the method $3U_{n+1} - 4U_n + U_{n-1} = 2hf(t_{n+1}, U_{n+1})$ (for approximating $u' = f(t,u)$) stable?

   b. Is it explicit or implicit?

c. Calculate the truncation error. (Hint: put in normalized form first.)

3. A certain quadrature method gives the following estimates of an integral:

h	I_h
0.125	42.0642089572
0.0625	42.0699513233
0.03125	42.0703214561

Estimate the order of convergence (without knowing the true value of the integral).

4. Estimate u(1.1) by taking one step of the Taylor series of order three (involving up to third derivatives in the Taylor series), with h=0.1, for the problem $\frac{du}{dt} = -tu, u(1) = 2$.

5. Write the third order equation:

$$u''' - sin(u'') + e^t u' + 2t\ cos(u) = 25$$
$$u(0) = 5, u'(0) = 3, u''(0) = 7$$

as a system of three first order equations:

$$u_1' = f_1(t, u_1, u_2, u_3)$$
$$u_2' = f_2(t, u_1, u_2, u_3)$$
$$u_3' = f_3(t, u_1, u_2, u_3)$$

with

$$u_1(0) = A$$
$$u_2(0) = B$$
$$u_3(0) = C$$

That is, find f_1, f_2, f_3, A, B, C.