Math 5329, Test III

Name _____

1. a. Find r, s which make the quadrature formula below as high order as possible ($x_i = a + ih, h = (b-a)/N$):

   $\int_a^b f(x)dx \approx \sum_{i=1}^N \frac{h}{2}[f(x_{i-1} + rh) + f(x_{i-1} + sh)]$

   (Hint: how are r and s related, by symmetry?)

   b. With this choice for r, s, what is the global order of this rule?

2. a. Is the following method stable? (Justify answer)

$$\frac{U_{k+1}-U_{k-2}}{3h} = \frac{1}{2}f(t_k, U_k) + \frac{1}{2}f(t_{k-1}, U_{k-1})$$

b. (Extra credit) Find the truncation error, and tell if the method is consistent or not.

3. a. A quadrature method gives an error of 10^{-5} when $h = 10^{-2}$ and 10^{-11} when $h = 10^{-4}$. Estimate the order of the method.

b. A differential equation solver gives an answer $u(1) = 1.020$ when $h = 0.1$, and $u(1) = 1.004$ when $h = 0.05$, and $u(1) = 1.003$ when $h = 0.025$. Estimate the order of the method.

4. a. Write the third order differential equation $u''' - 3u'' - u = t^2$ as a system of three first order equations, that is, in the form:

$u' = f(t, u, v, w) =$

$v' = g(t, u, v, w) =$

$w' = h(t, u, v, w) =$

b. Now write out the formulas for $u_{n+1}, v_{n+1}, w_{n+1}$ for Euler's method applied to this system of first order equations:

$u_{n+1} =$

$v_{n+1} =$

$w_{n+1} =$

5. If the third order Taylor series method (two more terms than Euler's method) is used to solve $u' = t^2 + 5u$, write u_{n+1} in terms of h, t_n and u_n only. $(t_n = nh, u_n \approx u(t_n))$