Math 5330, Test I

Name ______________________________

1. If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 8 \\ -2 & -5 & -4 \end{bmatrix}$$

find a permutation matrix P, a lower triangular matrix L, and an upper triangular matrix U such that $PA = LU$.

2. Use the decomposition of A from Problem 1 to solve $Ax = b$, where $b = (8, 18, -16)$. That is, multiply both sides by $P: \; PAx = Pb$ so $LUx = Pb$, then solve $Ly = Pb$, then $Ux = y$.

3. Consider the linear system:

$$\begin{bmatrix} -4 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

a. If the Jacobi iterative method is written in the form $x^{n+1} = Bx^n + c$, what is B?

b. Determine *theoretically* if the Jacobi method will converge, without doing any actual iterations.

c. If the Gauss-Seidel method is written in the form $x^{n+1} = Bx^n + c$, what is B?

d. Determine *theoretically* if the Gauss-Seidel method will converge, without doing any actual iterations.

4. Approximately how many multiplications does the following MATLAB program do, for large N?

```
X = 1;
for L=1:N
   for I=1:L
      for J=1:I
        for K=1:J
           X = X*X;
        end
      end
   end
end
```

5. Compute the condition number (infinity norm) for each of these matrices and tell which you would expect to produce the most relative round-off error, using Gauss elimination with partial pivoting?

$$\begin{bmatrix} 1000 & 1001 \\ -999 & -1000 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 10^{-9} & 0 \\ 0 & 10^{9} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

6. Define:

a. orthogonal matrix

b. lower Hessenberg matrix

c. tridiagonal matrix

d. positive definite matrix

e. $\|x\|_\infty$, if x is a vector

f. $\|A\|_p$, if A is a matrix