

Math 5330, Test II

Name _____

1. a. Find a QR decomposition of

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$$

- b. Do one complete iteration of the QR method to the matrix A . Is the new matrix more nearly diagonal, in the sense that the sum of squares of the off-diagonal elements is smaller (note: sum of squares of entire matrix will be the same)?
- c. Use the Jacobi method to find all eigenvalue and eigenvectors of A . (Note: only one iteration is necessary!)

2. Prove that if z is a solution to $AA^T z = b$, then $x \equiv A^T z$ is a solution of $Ax = b$ of minimum norm.

3. Under certain conditions, the QR iteration produces a quasitriangular matrix in the limit.

a. Define a quasitriangular matrix.

b. In general terms, how do you find the eigenvalues of a quasitriangular matrix?

c. Is it necessary for convergence, to start the QR iteration from Hessenberg form? What is the advantage of starting from Hessenberg form?

- d. If A is symmetric, show that $B = Q^T A Q$ is still symmetric, if Q is an orthogonal matrix. This means that if the original matrix is symmetric, and orthogonal transformations are used to reduce it to upper Hessenberg form, the resulting matrix has what (non-zero) structure? Is $B = M^{-1} A M$ still symmetric, if M is not orthogonal?
- e. If A is upper Hessenberg, the work to do one QR iteration is proportional to what power of N (size of matrix)? What if A is tridiagonal and symmetric? What will happen if the QR iteration is applied to a matrix that is tridiagonal and **not** symmetric?
3. Consider the iteration $A_{n+1} = A A_n$, where $A_0 = A$, and assume A is diagonalizable ($A = P^{-1} D P$).
- a. Show that in the limit as $n \rightarrow \infty$, $A_{n+1} = \lambda_1 A_n$, where λ_1 is the largest eigenvalue of A in absolute value (assume there is a largest eigenvalue).
- b. The normal power iteration is $v_{n+1} = A v_n$, that is, we normally

start with a random vector v_0 and multiply it repeatedly by A , rather than start with the matrix A and multiply it repeatedly by A . What is the advantage of the normal power iteration compared to the alternative approach defined above? Is there any potential disadvantage?

- c. Can you think of a way in which the iteration $A_{n+1} = AA_n$ could be made more efficient? (Hint: suppose n is a power of 2)