

Math 5330, Test II

Name _____

1. a. Find $\min\|Ax-b\|_2$, using orthogonal reduction, where $b = (1, 1, -1)$ and:

$$A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \\ 0 & 12 \end{bmatrix}$$

- b. Find $\min\|Ax - b\|_2$ using the normal equations method.

2. Find the straight line $f(x) = mx + b$ which most nearly interpolates the points $(0, -2), (2, 1), (3, 2), (5, 7)$ in the least squares sense.

3. If

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$$

a. Do one QR iteration on A.

b. Use the Jacobi method to find all eigenvalues and eigenvectors of A. (Note: only one iteration is necessary!)

c. Use the power method to find the largest eigenvalue (in absolute value) of A, starting with $x_0 = (2, 1)$.

4. a. Show that if A is symmetric, and Q is orthogonal, QAQ^{-1} is still symmetric.

b. Find an orthogonal matrix Q such that QAQ^{-1} is upper Hessenberg (and therefore tridiagonal, since A is symmetric).

$$A = \begin{bmatrix} 1 & -12 & 5 \\ -12 & 2 & 3 \\ 5 & 3 & 4 \end{bmatrix}$$

5. Find the eigenvalues of the quasitriangular matrix:

$$A = \begin{bmatrix} 2 & -12 & 5 & -7 \\ 0 & 2 & -6 & 3 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$