Math 5330, Test II

Name ____________________________

1. a. In problem 2.9, you proved that if z is a solution to $AA^Tz = b$, then $x \equiv A^Tz$ is a solution of $Ax = b$ of minimum norm. Use this to find the quadratic polynomial $a + bx + cx^2$, with minimum $a^2 + b^2 + c^2$, which passes through the two points $(0, 2), (1, 3)$.

   b. Find the least squares quadratic polynomial fit, $a + bx + cx^2$, to the points $(0, 2), (1, 3), (2, 2), (3, 2)$. Use the normal equations, but you do not need to solve the final linear system.

2. What is the order of work (power of N) for each of the following? Assume all matrices are N by N and full unless otherwise stated, and assume advantage is taken of any special structure mentioned.

   a. The Jacobi method to find the eigenvalues of a symmetric matrix A.

b. One LR iteration, if A is upper Hessenberg (assume no pivoting).

c. One QR iteration, if A is symmetric and tridiagonal.

d. One power method iteration.

e. The first inverse power method iteration.

f. The second inverse power method iteration, assuming the LU decomposition from the first iteration is saved.

g. The orthogonal transformation of a full matrix to a similar upper Hessenberg matrix.

3. Explain how you would find the vector x which minimizes $\|Ax - b\|_2$, if you already have the QR decomposition of the M by N matrix A. The operation count would be $O(N^\alpha)$ for what α, if we assume $M \approx 2N$? What would the operation count be if you don't have a QR decomposition?

4. If

$$A = \begin{bmatrix} 5 & 0 & 3 \\ 0 & -1 & 0 \\ 3 & 0 & 5 \end{bmatrix}$$

a. Use the Jacobi method to find all eigenvalues of the matrix. Only one iteration is necessary!

b. Find an orthogonal matrix Q such that QAQ^T is upper Hessenberg (and quasi-triangular).

c. Now find the eigenvalues of the quasi-triangular matrix resulting from part (b).

d. Do 2 or 3 iterations of the power method to find the largest eigenvalue (in absolute value) of A, starting with $x_0 = (2, 0, 3)$.