Math 5330 Test III

Name ____________________________

`Solve any 5 of the 6 problems.`

1. Write a linear programming problem which, if solved (but don't solve it), would produce the straight line $y = mx + b$ which most closely fits the data points $(0, 1), (1, 5), (2, 5)$ in the L_∞ norm. Write the constraints in the form $Ax \geq b$. (Hint: you will have 3 unknowns, m, b and ϵ, and 3 constraints involving absolute values, which translate into 6 linear constraints.)

2. Two factories have 300 and 400 cars, three dealers need 100, 120 and 60 delivered to them. The cost C_{ij} to transport each car from factory i to dealer j is: $C_{11} = 100, C_{12} = 150, C_{13} = 180, C_{21} = 250, C_{22} = 240, C_{23} = 180$. Set up the initial simplex tableaux for this problem (but don't solve!), using slack and artificial variables as needed.

3. Use the simplex method to solve
max $P = 2x_1 + 4x_2 + x_3 + x_4$
with

$$\begin{aligned} 2x_1 + x_2 + 2x_3 + 3x_4 &\leq 12 \\ 2x_2 + x_3 + 2x_4 &\leq 20 \\ 2x_1 + x_2 + 4x_3 \quad &\leq 16 \end{aligned}$$

and $x_1, x_2, x_3, x_4 \geq 0$
(Hint: the final basis will consist of x_1, x_2, s_3, where s_3 is the third slack variable; you can use this information to save a lot of work if you want.)

4. Use the simplex method to solve
max $P = x_1 + x_2 + 2x_3$
with

$$\begin{aligned} x_1 + 2x_2 - x_3 &\leq 6 \\ 2x_1 + x_2 - x_3 &\leq 6 \end{aligned}$$

and $x_1, x_2, x_3 \geq 0$

5. Use the simplex method to solve
max $P = 3x_1 + 6x_2 + 10x_3$
with

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &\leq 400 \\ 2x_1 + x_2 + 2x_3 &\leq 350 \end{aligned}$$

and $x_1, x_2, x_3 \geq 0$

6. a. Find the (symmetric) dual of problem 5 and set up the *initial* simplex tableaux for this problem, complete with artificial variables. Do not solve. Which is easier to solve, the primal (a) or dual (b)? What is the minimum of the objective for the dual problem?

   b. The solution of the dual problem is $y = (2.5, 0)$. Knowing this, if the 400 on the right hand side of the first inequality in (a) were increased to 401 and the problem were re-solved, what what the new P be?