Math 5330 Final Exam

Name _____

1. What is the order of work for each of the following? Assume all matrices are N by N and full unless otherwise stated, and assume advantage is taken of any special structure mentioned.

   a. One iteration (knock out one element) of the Jacobi method to find the eigenvalues of a symmetric matrix A.

   b. Solution of $Ax = b$ using Gaussian elimination, if A is upper Hessenberg.

   c. One QR iteration, if A is full.

   d. One LR iteration, if A is upper Hessenberg.

   e. One QR iteration, if A is symmetric and tridiagonal.

   f. Reduction to upper Hessenberg form, using orthogonal similarity transformations.

   g. Solution of min $\|Ax - b\|_2$ using the normal equations, where A is M by N, and $M >> N$.

   h. Solution of min $\|Ax - b\|_2$ using orthogonal reduction, where A is M by N, and $M >> N$.

   i. One simplex step, for solving max $c^T x$ with $Ax \leq b, x \geq 0$, where A is M by N, and $N >> M$.

   j. Solution of $Ax = b$ if an LU decomposition is known.

   k. One iteration of the inverse power method, for finding the smallest eigenvalue of tridiagonal matrix A.

   l. Solution of $Ax = b$ using Gaussian elimination, if A is banded, with bandwidth $N^{\frac{1}{3}}$.

2. Use the simplex method to solve:

   $\max P = 3x + 4y$
   with

$$\begin{aligned} x + y &\leq 6 \\ 2x + y &\leq 8 \end{aligned}$$

   and $x, y \geq 0$

3. Write the (symmetric) dual to the previous problem, and set up the initial simplex tableaux, with slack and artificial variables.

4. Find the straight line $y = p + qx$ which most closely fits the data points $(0, 1), (1, 6), (2, 2)$ in the L_2 norm.

5. Find A, b, c such that the following LP problem, if solved, would produce the straight line which most closely fits the data points of problem 4 in the L_1 norm.

   minimize $b^T y$, with $A^T y \geq c$.

   Here $y = [p, q, \epsilon_1, \epsilon_2, \epsilon_3]$ is the vector of unknowns. (Note: the dual of this problem would be: maximize $c^T x$, with $Ax \leq b, x \geq 0$, which could actually be solved by the simplex method.)