Math 5330 Final

Name ____________________________

1. Use the simplex method to solve

   a. max $P = 3x_1 + 6x_2 + 10x_3$
   with

$$
\begin{aligned}
2x_1 + 3x_2 + 4x_3 &\leq 400 \\
2x_1 + x_2 + 2x_3 &\leq 350
\end{aligned}
$$

   and $x_1, x_2, x_3 \geq 0$

   b. Find the (symmetric) dual of this problem and solve it graphically.

c. What is the significance of y_1 (the first component of the dual solution), relative to the original problem?

2. Given that the primal problem:

$$\text{maximize } P = c^T x$$

with constraints $Ax = b$ and bounds $x \geq 0$ has dual:

$$\text{minimize } D = b^T y$$

with constraints $A^T y \geq c$

a. What is the dual of the primal problem:

$$\text{maximize } P = c^T x$$

with constraints

$$\begin{array}{ccccccc}
a_{1,1}x_1 & + & \dots & + & a_{1,N}x_N & \leq & b_1, \\
\vdots & & & & \vdots & & \vdots \\
a_{k,1}x_1 & + & \dots & + & a_{k,N}x_N & \leq & b_k, \\
a_{k+1,1}x_1 & + & \dots & + & a_{k+1,N}x_N & = & b_{k+1}, \\
\vdots & & & & \vdots & & \vdots \\
a_{M,1}x_1 & + & \dots & + & a_{M,N}x_N & = & b_M,
\end{array}$$

and bounds $x \geq 0$?

b. Show directly (without using the fact that the problems are duals) that the minimum of the dual problem is greater than or equal to the maximum of the primal problem (they are actually equal, but you don't need to show that). If the primal problem is unbounded (maximum is infinite) what can we say about the dual problem?

3. Prove that the Gauss–Seidel iteration:
$x_i^{n+1} = \frac{1}{a_{ii}}\left(b_i - \sum_{j<i} a_{ij}x_j^{n+1} - \sum_{j>i} a_{ij}x_j^n\right)$
converges when A is diagonal-dominant, that is, when $|a_{ii}| > \sum_{j\neq i}|a_{ij}|$, for each i. Note that the exact solution satisfies:
$x_i = \frac{1}{a_{ii}}\left(b_i - \sum_{j<i} a_{ij}x_j - \sum_{j>i} a_{ij}x_j\right)$

4. a. Write a linear programming problem which, if solved (but don't solve it), would produce the straight line $y = mx + b$ which most closely fits the data points $(0,1),(1,5),(2,5)$ in the L_1 norm. Write the constraints in the form $Ax \geq b$. (Hint: you will have 5 unknowns, $m, b, \epsilon_1, \epsilon_2, \epsilon_3$, and 3 constraints involving absolute values, which should be translated into 6 linear constraints.)

b. Find the line $y = mx + b$ which most closely fits these same data point in the L_2 norm.