Math 5330 Final Exam

Name ______________________________

1. a. Find the straight line $y = mx + b$ which most closely fits the data points $(0, 1), (1, 5), (2, 5)$ in the L_2 norm.

   b. Write a linear programming problem which, if solved (but don't solve it), would produce the straight line $y = mx + b$ which most closely fits these same data points in the L_1 norm. Write the constraints in the form $Ax \geq b$. (Hint: you will have 5 unknowns, $m, b, \epsilon_1, \epsilon_2, \epsilon_3$, and 3 constraints involving absolute values, which translate into 6 linear constraints.)

2. Use the simplex method to solve

a. max $P = 2x_1 + 4x_2 + x_3 + x_4$
with

$$\begin{aligned} 2x_1 + x_2 + 2x_3 + 3x_4 &\leq 12 \\ 2x_2 + x_3 + 2x_4 &\leq 20 \\ 2x_1 + x_2 + 4x_3 \qquad &\leq 16 \end{aligned}$$

and $x_1, x_2, x_3, x_4 \geq 0$
(Hint: the final basis will consist of x_1, x_2, s_3, where s_3 is the third slack variable; you can use this information to save a lot of work if you want.)

b. max $P = x_1 + 3x_2 + 2x_3$
with

$$\begin{aligned} x_1 - 2x_2 + x_3 &\leq 2 \\ 2x_1 \qquad + x_3 &\leq 6 \end{aligned}$$

and $x_1, x_2, x_3 \geq 0$

c. What is the dual problem of [b.] and what is its solution?

3. Two factories have 300 and 400 cars, three dealers need 100, 120 and 60 delivered to them. The cost C_{ij} to transport each car from factory i to dealer j is: $C_{11} = 100, C_{12} = 150, C_{13} = 180, C_{21} = 250, C_{22} = 240, C_{23} = 180$. Set up the initial simplex tableaux for this problem (but don't solve!), using slack and artificial variables as needed.

4. If

$$A = \begin{bmatrix} 1 & 9 & 3 \\ 9 & 2 & 3 \\ 3 & 3 & 4 \end{bmatrix}$$

a. Find an elementry matrix M such that MAM^{-1} is upper Hessenberg.

b. The original A is symmetric; is MAM^{-1} still symmetric, and therefore tridiagonal? Show that if Q is orthogonal, QAQ^{-1} would still be symmetric.

5. If

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -3 & -2 & 1 \\ 0 & 4 & 0 \end{bmatrix}$$

Do one complete LR iteration on A.

6. Find the eigenvalues of the quasitriangular matrix:

$$A = \begin{bmatrix} 0 & -1 & 5 & -7 \\ 1 & 0 & 5 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

7. If the power method applied to $(A - pI)^{-1}$ finds a largest eigenvalue of μ, what is the eigenvalue of A closest to p?

8. What is the order of work for each of the following? Assume all matrices are N by N and full unless otherwise stated, and assume advantage is taken of any special structure mentioned.

   a. One iteration of the Jacobi method to find the eigenvalues of a symmetric matrix A.

   b. Solution of $Ax = b$ using Gaussian elimination, if A is upper Hessenberg.

   c. One QR iteration, if A is symmetric and tridiagonal.

   d. One Simplex step, for solving max $c^T x$ with $Ax \leq b, x \geq 0$, where A is M by N.

   e. Solution of $Ax = b$ if an LU decomposition is known.

   f. One iteration of the inverse power method, for finding the smallest eigenvalue of tridiagonal matrix A.

   g. Solution of $Ax = b$ using Gaussian elimination, if A is banded, with bandwidth $\sqrt{N}$.

   h. Reduction of a full N by N matrix to a similar upper Hessenberg matrix, by pre and post-multiplications by orthogonal Givens matrices.