MATH 5343

Test I   Name___________________________

1. True or False:

   a. If the LU decomposition of a tridiagonal matrix is done using partial pivoting, the resulting matrices L and U are also tridiagonal.

   b. The backwards (implicit) Euler method
   $\frac{U_{k+1}-U_k}{h} = f(t_{k+1}, U_{k+1})$
   is much better suited for stiff problems than the forwards (explicit) Euler method.

   c. All Adams methods (for $u' = f(t, u)$) are stable.

   d. It is easier to vary the stepsize h for Runge-Kutta methods than for multistep methods.

   e. An equation of third degree ($u''' = f(t, u, u', u'')$) can be reduced to a system of three first order equations.

   f. The number of multiplications required to solve an N by N full system of linear equations using Gaussian elimination is $\frac{1}{3}N^2$.

   g. It is possible to design explicit multistep methods which work well on stiff systems.

   h. For stable methods, the error is of the same order as the truncation order.

   i. If an ODE solver produces an error of 10^{-3} at $t = 1$, when $h = 0.01$, and an error of 10^{-6} at $t = 1$, when $h = 0.001$, then the experimental global error is $O(h^2)$.

   j. The amount of work required to solve an N by N tridiagonal system using a band solver is $O(N)$.

2. a. Is the following method stable? (Justify answer)

   $$\frac{U_{k+1}+4U_k-5U_{k-1}}{6h} = \frac{2}{3}f(t_k, U_k) + \frac{1}{3}f(t_{k-1}, U_{k-1})$$

   b. Is the method consistent? (Justify answer)

   c. Is this a backward difference method?

   d. Is this an Adams method?

3. a. Find the optimal values for A and B in the approximation (to $u' = f(t, u)$)

$$\frac{U_{k+1} - U_k}{h} = Af(t_k, U_k) + Bf(t_{k-1}, U_{k-1})$$

b. What is then the truncation error?

c. Is the method implicit or explicit?

4. For the problem $u' = -100u, u(0) = 1$, which has exact solution $u(t) = e^{-100t}$,

a. Compute the Euler method approximation to $u(1)$ using $h = 0.1$. For what values of h does Euler's method produce a good approximation?

b. Compute the backward Euler method approximation to $u(1)$ using $h = 0.1$. For what values of h does backward Euler produce a good approximation?