MATH 5343
Test I Name__________________________

1. True or False:

   a. Once the LU decomposition of a matrix A is known, a system $Ax = b$ can be solved in $O(N^2)$ operations.
   b. All Adams methods (for $u' = f(t, u)$) are stable.
   c. When a Runge-Kutta method of high order is used, a different method must be used to take the first few steps.
   d. It is possible to design explicit multistep methods which work well on stiff systems.
   e. If an ODE solver produces an error of 10^{-3} at $t = 1$, when $h = 0.01$, and an error of 10^{-7} at $t = 1$, when $h = 0.001$, then the experimental global error is $O(h^4)$.
   f. If a matrix is diagonal dominant and symmetric, then partial pivoting means no pivoting.
   g. If an N by N band matrix A has half-bandwidth L, where $1 \ll L \ll N$, a system $Ax = b$ can be solved in $O(NL)$ operations using a band solver.
   h. When using a multistep method of $O(h^4)$ truncation error, and a Taylor series method to take the first few steps, a Taylor polynomial of degree at least 4 should be used, to preserve the order of the multistep method.
   i. If you need to solve several linear systems with the same *banded* matrix, but different right hand sides, it is more efficient to save the LU decomposition when solving the first system, and use this to solve the other systems, than to find the inverse and multiply each right hand side by the inverse matrix.

2. a. Is the following method stable? (Justify answer)

   $$\frac{U_{k+1} - U_{k-2}}{3h} = \frac{1}{2} f(t_k, U_k) + \frac{1}{2} f(t_{k-1}, U_{k-1})$$

   b. Find the truncation error, and tell if the method is consistent or not.

3. a. Find the optimal values for A and B in the approximation (to $u' = f(t,u)$)

$$\frac{U_{k+1} - U_k}{h} = Af(t_{k+1}, U_{k+1}) + Bf(t_k, U_k)$$

b. What is then the truncation error?

c. Is the method implicit or explicit?

4. The problem $u' = -au, u(0) = 1$, where $a > 0$, has exact solution $u(t) = e^{-at}$, which decreases with time.

a. For what (positive) values of h does the Euler method produce a solution which decreases (in absolute value) with time?

b. Same question as (a) for backward Euler method.

c. Same question as (a) for the method of problem 3.