MATH 5343

Test I Name___________________________

1. True or False:

   a. If Gaussian elimination is done on a large symmetric matrix and no pivoting is done, the work can be cut by approximately half by taking advantage of symmetry.
   b. Partial pivoting is equivalent to no pivoting, if the matrix is diagonal dominant and symmetric.
   c. All backward difference methods are stable.
   d. If two linear systems are solved with the same banded matrix, of size N by N and bandwidth k ($1 << k << N$), using Gaussian elimination, the first requires $O(Nk^2)$ work, but if the LU decomposition is saved, the second requires only $O(Nk)$ work.
   e. If an ODE solver produces an error of 10^{-6} at $t = 1$, when $h = 0.1$, and an error of 10^{-12} at $t = 1$, when $h = 0.001$, then the experimental global error is $O(h^2)$.
   f. Adaptive methods for ODEs decide on a stepsize by trying two different methods, or two different stepsizes, and comparing the results each step.
   g. It is possible to design explicit multistep methods which work well on stiff systems.
   h. The ODE system $y' = v, v' = -10000y$ is stiff.
   i. Forward Euler is better suited for stiff ODE problems than backward Euler.
   j. If you need to solve several systems with the same band matrix, it is more efficient to form the inverse and multiply each right hand side by the inverse, than to use the LU decomposition.

2. a. Is the following method stable? (Justify answer)

$$2U_{k+1} + 3U_k - 6U_{k-1} + U_{k-2} = 6hf(t_k, U_k)$$

b. Is the method consistent? (Justify answer, but don't necessarily have to find the truncation error.)

c. Is the method explicit?

3. a. Find the optimal values for A, B, C in the approximation (to $u' = f(t, u)$)

$$\frac{U_{k+1}-U_{k-1}}{2h} = Af(t_{k+1}, U_{k+1}) + Bf(t_k, U_k) + Cf(t_{k-1}, U_{k-1})$$

b. Is the method stable (Justify answer)?

4. Consider the general multistep method:

$$\frac{U(t_{k+1}) + \alpha_1 U(t_k) + \alpha_2 U(t_{k-1}) + \cdots + \alpha_m U(t_{k+1-m})}{h}$$

$$= \beta_0 f(t_{k+1}, U(t_{k+1})) + \cdots + \beta_m f(t_{k+1-m}, U(t_{k+1-m}))$$

a. Explain how the coefficients α_i, β_i are found for the Adams-Bashforth (explicit) methods, in general terms.

b. Explain how the coefficients α_i, β_i are found for the Adams-Moulton (implicit) methods.

c. Explain how the coefficients α_i, β_i are found for the backward difference methods.