MATH 5343

Final Name__________________________

1. We want to solve the PDE $-\nabla^2 U + U = f(x,y)$, with $\frac{\partial U}{\partial n} = -U + g(x,y)$ on the boundary. If a Galerkin finite element method is used, and U is expanded as a linear combination of basis functions $U(x,y) = \sum_{i=1}^{N} a_i \phi_i(x,y)$, a linear system of the form $A\mathbf{a} = \mathbf{b}$ will result. What are the elements A_{ki}, b_k of matrix A and vector $\mathbf{b}$. How many continuous derivatives must $\phi_i(x,y)$ have? Is A symmetric? Is A positive definite?

2. We want to solve the PDE $-\nabla^2 U + U = f(x,y)$, with $U = 0$ on the boundary. If a collocation finite element method is used, and U is expanded as a linear combination of basis functions $U(x,y) = \sum_{i=1}^{N} a_i \phi_i(x,y)$, where $\phi_i(x,y) = 0$ on the boundary, a linear system of the form $A\mathbf{a} = \mathbf{b}$ will result. What are the elements A_{ki}, b_k of matrix A and vector $\mathbf{b}$. How many continuous derivatives must $\phi_i(x,y)$ have? Is A symmetric?

3. Give two important advantages of the finite element method over finite difference methods.

4. Write out, in an form where no inverses appear, the shifted inverse power iteration for finding the eigenvalue of $A\mathbf{a} = \lambda B\mathbf{a}$ closest to p, and tell how this eigenvalue λ can be found from two consecutive iterates, z_n and z_{n+1}, after convergence.

5. Is the following approximation consistent with $u' = f(t, u)$, and is it stable? What is the order $O(h^\alpha)$ of the error at a fixed t?
$\frac{U(t_{k+1})-U(t_{k-2})}{3h} = \frac{1}{2}f(t_k, U(t_k)) + \frac{1}{2}f(t_{k-1}, U(t_{k-1}))$.

6. a. Under what conditions is the following method (to approximate $u_{tt} = u_{xx}$) stable, where $U_i^k \equiv U(x_i, t_k)$ (justify answer)? (Hint: $e^{Imdx} - 2 + e^{-Imdx} = -4sin^2(m * dx/2)$)

$$\frac{U_i^{k+1}-2U_i^k+U_i^{k-1}}{dt^2} = \frac{1}{3}\frac{U_{i+1}^{k-1}-2U_i^{k-1}+U_{i-1}^{k-1}}{dx^2} + \frac{1}{3}\frac{U_{i+1}^{k}-2U_i^{k}+U_{i-1}^{k}}{dx^2} + \frac{1}{3}\frac{U_{i+1}^{k+1}-2U_i^{k+1}+U_{i-1}^{k+1}}{dx^2}$$

b. What is the order $O(dt^n)+O(dx^m)$ of the truncation error? You can just guess!