CPS 5320 Theory-Based Qualifier Exam

09:00 am – 10:30 am on January 16, 2020

Name: ____

Student ID #: ______________________________________

Please read the following instructions carefully

1. This is a closed book exam.
2. The total time for this exam is 1 hour 30 minutes.
3. The exam is worth a total of 100 points.
4. You are permitted to use a simple (non-graphing, non-programmable) calculator. Cell phones, laptops, and all other web-enabled devices are not allowed.
5. Show sufficient work for full credit.

Number	Maximum Points	Earned Points
I	20	
II	10	
III	15	
IV	15	
V	20	
VI	20	
Total	100	

I. At low Reynold's number the 2D Navier Stokes equations reduce to:

$$f1 + \mu(U_{xx} + U_{yy}) = \rho U_t + P_x$$
$$f2 + \mu(V_{xx} + V_{yy}) = \rho V_t + P_y$$
$$U_x + V_y = 0$$

where $(U(x,y,t), V(x,y,t))$ is the fluid velocity vector, and $\mu, \rho, P(x,y,t)$ are the fluid viscosity, density and pressure, and $(f1(x,y,t), f2(x,y,t))$ is the external force field vector.

If we define a stream function $\phi(x,y,t)$ such that $(U,V) = (\phi_y, -\phi_x)$, show that the last (divergence) equation is automatically satisfied, and find the "stream function" formulation which consists of a system of two second order equations involving only ϕ and the "vorticity" $\omega \equiv U_y - V_x$. (Hint: eliminate P from the first two equations.)

II. If 99% of a program is parallelizable, what is the highest speed-up factor that could be expected when going from 1 to 16 processors? (Assume the communication time is negligible).

III. Consider the heat equation $\rho C_p T_t = \nabla \bullet [\kappa \nabla \mathbf{T} - \rho C_p T \mathbf{v}] + q$

a. Here $T(x, y, z, t)$, ρ and C_p represent temperature, density and specific heat of the medium. What do $\kappa(x, y, z)$, $\mathbf{v}(x, y, z)$, and $q(x, y, z, t)$ represent physically?

b. If $\kappa(x, y, z)$ is a discontinuous function, which finite element method, Galerkin or collocation, is better able to handle this case, and why?

c. If $\kappa > 0$, the temperature can be specified on the entire boundary. If $\kappa = 0$, on what part of the boundary could the temperature be specified?

IV. To use PDE2D to solve a PDE in the 3D region above $z = 0$ and below $z = 4 - x^2 - y^2$, you need to describe this paraboloid as $(X(p1, p2, p3), Y(p1, p2, p3), Z(p1, p2, p3))$, with constant limits on the parameters $p1, p2, p3$. Give a possible parameterization, with limits, of this region.

V. To derive the minimal surface equation, suppose $u(x, y)$ is the surface with $u = g(x, y)$ on the boundary $\partial\Omega$ of Ω which minimizes the surface area $SA(u) \equiv \int\int_\Omega \sqrt{1 + u_x^2 + u_y^2}\, dA$. Then if $e(x, y)$ is any smooth function with $e = 0$ on the boundary, $SA(u+\alpha e) \geq SA(u)$ for any α. From this we conclude that $f(\alpha) \equiv SA(u+\alpha e)$ has a minimum at $\alpha = 0$, and thus $f'(0) = 0$. Write out the equation $f'(0) = 0$ and then derive a partial differential equation for u.

Hint: you can use the multidimensional integration by parts formula:

$$\iint_\Omega \nabla w \bullet \mathbf{v} = \int_{\partial\Omega} w\mathbf{v} \bullet n - \iint_\Omega w\nabla \bullet \mathbf{v}$$

where w is a scalar function, $\mathbf{v}$ is a vector function.

VI. The subroutine MMUL below computes C=A*B, where A is a matrix and B is a vector, and only one processor is used (NPES=1).

```
      SUBROUTINE MMUL(A,B,C,N,NPES,ITASK)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION A(N,*),B(N),C(N),CI(N)
      include 'mpif.h'
      DO I=1,N
         CI(I) = 0
         DO J=1,N
            CI(I) = CI(I) + A(I,J)*B(J)
         ENDDO
      ENDDO
      CALL MPI_ALLREDUCE(CI,C,N,MPI_DOUBLE_PRECISION,
     &           MPI_SUM,MPI_COMM_WORLD,IERR)
      RETURN
      END
```

a. Modify MMUL, by changing only one line, so that it runs efficiently on NPES processors, if the columns of A are stored cyclically, 0,1,2,...NPES-1,0,1,2,...NPES-1,0,1,2... but each processor actually stores the whole matrix, that is, A is dimensioned A(N,N) in the calling program. Write the modified line below:

b. Changing only one more line, modify MMUL so that each processor only stores its columns, that is, A can be dimensioned A(N,(N-1)/NPES+1) (using ALLOCATE) in the calling program. Write the modified line below: