CPS 5320 Practice Test

Name ______________________________

1. If U is a function of $r = \sqrt{x^2+y^2}$ only, use the chain rule to express $U_{xx} + U_{yy}$ in terms of U_{rr}, U_r only.

   answer: $U_{rr} + U_r/r$

2. To derive the beam bending equation, suppose $u(x)$ is the height of the beam with $u(0) = g_0, u'(0) = h_0, u(L) = g_1, u'(L) = h_1$, which minimizes the energy $E(u) \equiv \int_0^L \frac{1}{2}D(u'')^2 - uq\ dx$, where $D(x)$ is the bending stiffness and $q(x)$ is an external vertical force. Then if $e(x)$ is any smooth function with $e(0) = e'(0) = e(L) = e'(L) = 0$ on the boundary, $E(u + \alpha e) \geq E(u)$ for any α. From this we conclude that $f(\alpha) \equiv E(u + \alpha e)$ has a minimum at $\alpha = 0$, and thus $f'(0) = 0$. Write out the equation $f'(0) = 0$ and explain how this equation can be used to find a differential equation for u. (You don't need to actually derive the differential equation, just outline what needs to be done.)

   answer: $\frac{df}{d\alpha}(0) = \int_0^L Du''e'' - eq\ dx = 0$. Do two integrations by parts to get $\int_0^L e[...]dx = 0$, since e is arbitrary, the expression in brackets must be 0.

3. Suppose we use Gauss elimination to solve an N by N linear system, and distribute the columns by "blocks" over the processors: each processor stores the entire matrix, but the first NB=N/NPES columns are only touched by processor 0, the second NB columns by processor 1, etc. How would you modify the innermost loop of DLINEQ, shown below:

```
      DO 25 K=I,N
         A(J,K) = A(J,K) - LJI*A(I,K)
   25 CONTINUE
```

Use ITASK for the processor number, and NPES for the number of processors, and assume N is divisible by NPES. Hint: the limits are simpler now than when the columns are distributed cyclically, and remember that if $I1 > I2$, no trips through a loop "DO K=I1,I2" will be made.

answer:

```
      DO 25 K=max(I,ITASK*NB+1),ITASK*NB+NB
         A(J,K) = A(J,K) - LJI*A(I,K)
   25 CONTINUE
```

4. What is the output from the MPI Fortran program below, if run on NPES=3 processors?

```
      PARAMETER (N=12)
      DOUBLE PRECISION X(N),SUMI,SUM
      INCLUDE 'mpif.h'
C                        INITIALIZE MPI
      CALL MPI_INIT (IERR)
C                        NPES = NUMBER OF PROCESSORS
      CALL MPI_COMM_SIZE (MPI_COMM_WORLD,NPES,IERR)
C                        ITASK = MY PROCESSOR NUMBER
```

```
      CALL MPI_COMM_RANK (MPI_COMM_WORLD,ITASK,IERR)
      DO I=1,N
         IF (MOD(I-1,NPES).EQ.ITASK) THEN
            X(I) = I
         ENDIF
      ENDDO
C
      SUMI = 0
      DO I=ITASK+1,N,NPES
         SUMI = SUMI + X(I)
      ENDDO
      iroot = 1
      CALL MPI_REDUCE(SUMI,SUM,1,MPI_DOUBLE_PRECISION,
     &                 MPI_SUM,iroot,MPI_COMM_WORLD,IERR)
      if (ITASK.EQ.iroot) PRINT *, SUMI,SUM
      CALL MPI_FINALIZE(IERR)
      STOP
      END
```

answer: 26,78

5. In deriving the Black-Scholes partial differential equation, we assumed what type of probability distribution for the price S at future time=T of an asset whose price is s at time=t? What did we assume for the mean and the standard deviation of this distribution? The asset volatility is σ_1, the strike price is E and the risk-free interest rate (rate of inflation, sort of) is r.

   answer: log normal distribution with mean $se^{r(T-t)}$ and standard deviation $\sigma = \sigma_1\sqrt{T-t}$.

6. Consider the diffusion partial differential equation:

$$C_t = \nabla \bullet [D\nabla\mathbf{C} - C\mathbf{v}] + q$$

   a. What do $D, \mathbf{v}$, and q represent physically?
      answer: D = diffusion coefficient, $\mathbf{v}$ = convection velocity field, q = generation rate due to sources/sinks.

   b. Do solutions tend to be smoother when $D = 0$?
      answer: no!

   c. If $D(x, y)$ is a discontinuous function, for example, if it jumps from one value in one subregion to another in another subregion, which finite element method, Galerkin or collocation, is better able to handle this case, and why? Will the density C be continuous then? How about ∇C?

      answer: Galerkin is better, for collocation we would have to write as $C_t = D\nabla^2 C + \nabla D \bullet \nabla C$... and ∇D is infinite at interface. C is continuous, but ∇C is not.