Can "ANYTHING" Happen in an Open System?

Appendix D in
"The Numerical Solution of Ordinary and Partial Differential Equations"
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The partial differential equations that govern heat conduction and diffusion, derived in Section 2.0 and studied extensively in this book, have some relevance to an interesting philosophical question.

Consider the diffusion (conduction) of heat in a solid, R, with (absolute) temperature distribution $U(x, y, z, t)$. The first law of thermodynamics (conservation of energy) requires that

$$Q_t = -\nabla^T \mathbf{J}, \qquad \text{(D.1)}$$

where Q ($Q = c\rho U$) is the heat energy density and $\mathbf{J}$ is the heat flux vector. The second law requires that the flux be in a direction in which the temperature is decreasing, that is,

$$\mathbf{J}^T \nabla U \leq 0 \qquad \text{(D.2)}$$

In fact, in an isotropic solid, $\mathbf{J}$ is in the direction of greatest decrease of temperature, that is, $\mathbf{J} = -K\ \nabla U$ (cf. equation 2.0.2). Note that D.2 simply says that heat flows from hot to cold regions—because the laws of probability favor a more uniform distribution of heat energy.

"Thermal entropy" is a quantity that is used to measure randomness in the distribution of heat. The rate of change of thermal entropy, S, is given by the usual definition (see Problem 10 of Chapter 2) as

$$S_t = \iiint\limits_R \frac{Q_t}{U} dV. \qquad \text{(D.3)}$$

Using D.3 and the first law D.1, we get

$$S_t = \iiint\limits_R \frac{-\mathbf{J}^T\ \nabla U}{U^2} dV - \iint\limits_{\partial R} \frac{\mathbf{J}^T \mathbf{n}}{U} dA, \qquad \text{(D.4)}$$

where $\mathbf{n}$ is the outward unit normal on the boundary ∂R. From the second law (D.2), we see that the volume integral is nonnegative, and so

$$S_t \geq -\iint_{\partial R} \frac{\mathbf{J}^T\mathbf{n}}{U} dA. \tag{D.5}$$

From D.5 it follows that $S_t \geq 0$ in an isolated, closed system, where there is no heat flux through the boundary ($\mathbf{J}^T\mathbf{n} = 0$). Hence, in a closed system, entropy can never decrease. Since thermal entropy measures randomness (disorder) in the distribution of heat, its opposite (negative) can be referred to as "thermal order," and we can say that the thermal order can never increase in a closed system.

Furthermore, there is really nothing special about "thermal" entropy. We can define another entropy, and another order, in exactly the same way, to measure randomness in the distribution of any other substance that diffuses; for example, we can let $U(x, y, z, t)$ represent the concentration of carbon diffusing in a solid (Q is just U now), and through an identical analysis show that the "carbon order" thus defined cannot increase in a closed system. It is a well-known prediction of the second law that, in a closed system, every type of order is unstable and must eventually decrease, as everything tends toward more probable (more random) states: Not only will carbon and temperature distributions become more random (more uniform), but the performance of all electronic devices will deteriorate, not improve. Natural forces, such as corrosion, erosion, fire, and explosions, do not create order, they destroy it. The second law is all about probability; it uses probability at the microscopic level to predict macroscopic change: The reason carbon distributes itself more and more uniformly in an insulated solid is, that is what the laws of probability predict, when diffusion alone is operative. The reason natural forces may turn a spaceship, or a TV set, or a computer into a pile of rubble but not vice versa is also probability. Of all the possible arrangements atoms could take, only a very small percentage could fly to the moon and back, or receive pictures and sound from the other side of the Earth, or add, subtract, multiply, and divide real numbers with high accuracy.

The discovery that life on Earth developed through evolutionary "steps", coupled with the observation that mutations and natural selection—like other natural forces—can cause (minor) change, is widely accepted in the scientific world as proof that natural selection—alone among all natural forces—can create order out of disorder, and even design human brains with human consciousness. Only the layman seems to see the problem with this logic. In a recent *Mathematical Intelligencer* article [Sewell 2000], after outlining the specific reasons why it is not reasonable to attribute the major steps in the development of life to natural selection, I asserted that the idea that the four fundamental forces of physics alone could rearrange the fundamental particles of Nature into spaceships, nuclear power plants, and computers, connected to laser printers, CRTs, keyboards, and the Internet, appears to violate the second law of thermodynamics in a spectacular way. Anyone who has made such an argument is familiar with the standard reply: The Earth is an open system, and order can increase in an open system, as long as it is "compensated" somehow by a comparable or greater decrease outside the system. S. Angrist and L. Hepler, for example, in *Order and Chaos* [Angrist and Hepler 1967], write: "In a certain sense the development of civilization may appear contradictory to the second law... Even though society can effect local reductions in entropy, the general and universal trend of entropy increase easily swamps the anomalous but important efforts of civilized man. Each localized, man-made or machine-made entropy decrease is accompanied by a greater increase in entropy of the surroundings, thereby maintaining the required increase in total entropy."

According to this reasoning, then, the second law does not prevent scrap metal from reorganizing itself into a computer in one room, as long as two computers in the next room are rusting into scrap metal—and the door is open. A closer look at equation D.5, which holds not only for thermal entropy but for the "entropy" associated with any other substance that diffuses, shows that this argument, which goes unchallenged in the scientific literature, is based on a misunderstanding of the second law. Equation D.5 does not simply say that entropy cannot decrease in a closed system, it also says that in an open system, entropy cannot decrease faster than it is exported through the boundary, because the boundary integral there represents the rate that entropy is exported across the boundary: Notice that the integrand is the outward heat flux divided by absolute temperature. (That this boundary integral represents the rate that entropy is exported seems to have been noticed by relatively few people [e.g., Dixon 1975, p. 202], probably because the isotropic case is usually assumed and so the numerator is written as $-K\frac{\partial U}{\partial n}$, and in this form the conclusion is not as obvious.) Stated another way, the order in an open system cannot increase faster than it is imported through the boundary. According to D.4, the thermal order in a system can decrease in two different ways—it can be converted to disorder (first integral term) or it can be exported through the boundary (boundary integral term). It can increase in only one way: by importation through the boundary. Similarly, the increase in "carbon order" in an open system cannot be greater than the carbon order imported through the boundary, and the increase in "chromium order" cannot be greater than the chromium order imported through the boundary, and so on.

The above analysis was published in my reply "Can ANYTHING Happen in an Open System?" [Sewell 2001] to critics of my original *Mathematical Intelli-*

gencer article. In these simple examples, I assumed nothing but heat conduction or diffusion was going on; but for more general situations, I offered the tautology that "*if an increase in order is extremely improbable when a system is closed, it is still extremely improbable when the system is open, unless something is entering which makes it **not** extremely improbable.*" The fact that order is disappearing in the next room does not make it any easier for computers to appear in our room—unless this order is disappearing **into** our room, and then only if it is a type of order that makes the appearance of computers not extremely improbable, for example, computers. Importing thermal order will make the temperature distribution less random, and importing carbon order will make the carbon distribution less random, but neither makes the formation of computers more probable. What happens in a closed system depends on the initial conditions; what happens in an open system depends on the boundary conditions as well.

As I wrote in Sewell [2001], "order can increase in an open system, not because the laws of probability are suspended when the door is open, but simply because order may walk in through the door...If we found evidence that DNA, auto parts, computer chips, and books entered through the Earth's atmosphere at some time in the past, then perhaps the appearance of humans, cars, computers, and encyclopedias on a previously barren planet could be explained without postulating a violation of the second law here (it would have been violated somewhere else!). But if all we see entering is radiation and meteorite fragments, it seems clear that what is entering through the boundary cannot explain the increase in order observed here."