

## Review of Multivariate Calculus

In what follows,  $u, w$  represent scalar functions and  $\mathbf{v} = (v_1, v_2, v_3)$  represents a vector function.  $n$  represents the unit outward normal vector, to the boundary  $\partial R$ .

$$\text{gradient } u \equiv \nabla u = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) u = \left( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z} \right)$$

$$\text{divergence } \mathbf{v} \equiv \nabla \cdot \mathbf{v} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \mathbf{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

$$\nabla \cdot \nabla u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \equiv \nabla^2 u$$

Divergence theorem:

$$\iiint_R \nabla \cdot \mathbf{v} = \iint_{\partial R} \mathbf{v} \cdot \mathbf{n}$$

Product rules (second is just first with  $\mathbf{v} = \nabla w$ )

$$\nabla \cdot (u\mathbf{v}) = u\nabla \cdot \mathbf{v} + \nabla u \cdot \mathbf{v}$$

$$\nabla \cdot (u\nabla w) = u\nabla^2 w + \nabla u \cdot \nabla w$$

Integration by parts (follow from product rules and divergence theorem):

$$\iiint_R u\nabla \cdot \mathbf{v} = \iint_{\partial R} u\mathbf{v} \cdot \mathbf{n} - \iiint_R \nabla u \cdot \mathbf{v}$$

$$\iiint_R u\nabla^2 w = \iint_{\partial R} u\nabla w \cdot \mathbf{n} - \iiint_R \nabla u \cdot \nabla w = \iint_{\partial R} u \frac{\partial w}{\partial n} - \iiint_R \nabla u \cdot \nabla w$$