2D Eigenvalue Problems (Galerkin method)

PDEs (must be linear):

$$
\begin{aligned}
&\frac{\partial A_1}{\partial x}(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
&+\frac{\partial B_1}{\partial y}(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y) &=&\ F_1(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
& &&+\lambda\rho_{11}(x,y)U1+...+\lambda\rho_{1N}(x,y)UN \\
& &=& \\
& &=& \\
&\frac{\partial A_N}{\partial x}(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
&+\frac{\partial B_N}{\partial y}(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y) &=&\ F_N(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
& &&+\lambda\rho_{N1}(x,y)U1+...+\lambda\rho_{NN}(x,y)UN
\end{aligned}
$$

Boundary conditions:

$$
\begin{aligned}
U1 &= FB_1(x,y) \\
&= \\
&= \\
UN &= FB_N(x,y)
\end{aligned}
$$

or

$$
\begin{aligned}
A_1N_x+B_1N_y &= GB_1(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
&= \\
&= \\
A_NN_x+B_NN_y &= GB_N(x,y,U1,U1_x,U1_y,...,UN,UN_x,UN_y)
\end{aligned}
$$

where (N_x,N_y) = unit outward normal vector.