# 2D Time-Dependent Problems (Galerkin method)

PDEs:

$$
\begin{aligned}
C_{11}\frac{\partial U1}{\partial t} + \cdots + C_{1N}\frac{\partial UN}{\partial t} &= \frac{\partial}{\partial x}A_1(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
&\quad + \frac{\partial}{\partial y}B_1(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
&\quad - F_1(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
. &= . \\
. &= . \\
C_{N1}\frac{\partial U1}{\partial t} + \cdots + C_{NN}\frac{\partial UN}{\partial t} &= \frac{\partial}{\partial x}A_N(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
&\quad + \frac{\partial}{\partial y}B_N(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
&\quad - F_N(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y)
\end{aligned}
$$

where the $C_{ij}$ are functions of $(x,y,t,U1,...,UN)$.

Boundary conditions:

$$
\begin{aligned}
U1 &= FB_1(x,y,t) \\
. &= . \\
. &= . \\
UN &= FB_N(x,y,t)
\end{aligned}
$$

or

$$
\begin{aligned}
A_1N_x + B_1N_y &= GB_1(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y) \\
. &= . \\
. &= . \\
A_NN_x + B_NN_y &= GB_N(x,y,t,U1,U1_x,U1_y,...,UN,UN_x,UN_y)
\end{aligned}
$$

where $(N_x, N_y)$ = unit outward normal vector.

Initial conditions:

$$
U1(x,y,t_0) = U1_0(x,y)
$$

$$
\begin{aligned}
. &= . \\
. &= . \\
UN(x, y, t_0) &= UN_0(x, y)
\end{aligned}
$$