2D Time-Dependent Problems (Galerkin method)

PDEs:

$$
\begin{aligned}
C_{11}\frac{\partial U1}{\partial t} + ... + C_{1N}\frac{\partial UN}{\partial t} &= \frac{\partial A_1}{\partial x}(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) \\
&+ \frac{\partial B_1}{\partial y}(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) \\
&- F_1(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) \\
&= \\
&= \\
C_{N1}\frac{\partial U1}{\partial t} + ... + C_{NN}\frac{\partial UN}{\partial t} &= \frac{\partial A_N}{\partial x}(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) \\
&+ \frac{\partial B_N}{\partial y}(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) \\
&- F_N(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y)
\end{aligned}
$$

where the C_{ij} are functions of $(x, y, t, U1, ..., UN)$.

Boundary conditions:

$$
\begin{aligned}
U1 &= FB_1(x, y, t) \\
&= \\
&= \\
UN &= FB_N(x, y, t)
\end{aligned}
$$

or ((N_x, N_y) = unit outward normal vector):

$$
\begin{aligned}
A_1 N_x + B_1 N_y &= GB_1(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) \\
&= \\
&= \\
A_N N_x + B_N N_y &= GB_N(x, y, t, U1, U1_x, U1_y, ..., UN, UN_x, UN_y)
\end{aligned}
$$

Initial conditions:

$$
\begin{aligned}
U1(x, y, t_0) &= U1_0(x, y) \\
&= \\
&= \\
UN(x, y, t_0) &= UN_0(x, y)
\end{aligned}
$$