The explicit, upwind approximation to $u_t = -vu_x$ ($v > 0$) is:

$$\frac{U(x_i, t_{k+1}) - U(x_i, t_k)}{\Delta t} = -v\frac{U(x_i, t_k) - U(x_{i-1}, t_k)}{\Delta x}$$

We substitute $U(x_i, t_k) = a_m(t_k)e^{Imx_i}$:

$$\frac{a_m(t_{k+1})e^{Imx_i} - a_m(t_k)e^{Imx_i}}{\Delta t} = -v\frac{a_m(t_k)e^{Imx_i} - a_m(t_k)e^{Imx_{i-1}}}{\Delta x}$$

or

$$\frac{a_m(t_{k+1}) - a_m(t_k)}{\Delta t}e^{Imx_i} = -va_m(t_k)\frac{e^{Imx_i} - e^{Im(x_i - \Delta x)}}{\Delta x}$$

Dividing through by e^{Imx_i}:

$$\frac{a_m(t_{k+1}) - a_m(t_k)}{\Delta t} = -va_m(t_k)\frac{1 - e^{-Im\Delta x}}{\Delta x}$$

or:

$$a_m(t_{k+1}) - a_m(t_k) = -r(1 - e^{-Im\Delta x})a_m(t_k)$$

where $r \equiv \frac{v\Delta t}{\Delta x}$. Then the characteristic polynomial is:

$$\lambda + (-1 + r(1 - e^{-Im\Delta x})) = 0$$

which has the single root $\lambda = (1 - r) + re^{-Im\Delta x}$. As m varies from 1 to N, λ traces out a circle in the complex plane with center at $1 - r$ and radius r. Geometrically, it is clear that all these points will lie inside the complex circle $|\lambda| = 1$, if and only if the radius $r \leq 1$, that is, if $\Delta t \leq \frac{\Delta x}{v}$. Thus the explicit upwind method is stable if and only if this bound on Δt holds.