Oct 3 Homework

1. In order to better understand Schumer's proof for problem 6.2, consider the case n=10 and write $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{10}$ as a single fraction, with denominator equal to the lowest common multiple of the numbers 1,2,3,...,10. Note that the denominator is even, and all terms in the numerator are also even except one. Which one is odd? Explain why.

2. Using a Taylor series expansion for $ln(1-x)$, show that

   $\int_0^1 -\frac{ln(1-x)}{x}dx = \sum_{i=1}^{\infty} \frac{1}{i^2}$

   Then use some technology (MATLAB, Maple,...) to approximately evaluate this integral, and verify it is about $\pi^2/6$.

3. Get upper and lower bounds on $\sum_{i=1}^{\infty} \frac{1}{i^2}$ by adding the first 10 terms, then using integrals to get upper and lower bounds on the rest of this series.

4. Prove that if f(x) is a monotone nonincreasing, nonnegative function, and $f(2)$ is finite, then $\sum_{i=2}^{\infty} f(i)$ is finite if and only if $\int_2^{\infty} f(x)dx$ is finite.

5. Use problem 4 to determine if $\sum_{i=2}^{\infty} f(i)$ is finite, where:

   a. $f(i) = 1/i$
   b. $f(i) = 1/i^{1.00001}$
   c. $f(i) = 1/(i * ln(i))$
   d. $f(i) = 1/(i * (ln(i))^{1.00001})$

   (We are getting pretty close to the boundary between convergence and non-convergence!)

6. a. Show that the series in problem 5c diverges using a proof similar to the first proof of Proposition 6.1, that is, by looking at the series $\sum_{k=1}^{\infty} S_k$, where $S_1 = a_2$, $S_2 = a_3 + a_4$, $S_3 = a_5 + a_6 + a_7 + a_8$, etc., and getting a lower bound on S_k.

   b. Show that the series in problem 5d converges using a proof similar to that in problem 6a, but of course getting an upper bound on S_k. (Hint: You may use the fact that 5b converges).