Oct 10 Homework

1. If S is a subset of the positive integers, let's define $\mu(N)$= fraction of positive integers $\leq N$ that belong to S. If $\mu(N)$ converges to something greater than 0 as N goes to infinity, prove that $\sum_{i\ in\ S} \frac{1}{i}$ must be infinite.

2. If $\mu(N)$ (see problem 1) converges to 0, show by examples that $\sum_{i\ in\ S} \frac{1}{i}$ may be finite or infinite. Thus if the positive integers are "thinned out" more and more as we go to larger integers, this does not necessarily mean that the remaining terms of the harmonic series will converge. (Hint: see p143)

3. Find approximately the lowest N such that $\sum_{i=2}^{N} a_i > 100$, if (note all series are divergent):

   a. $a_i = \frac{1}{i}$
   b. $a_i = \frac{1}{i*ln(i)}$
   c. $a_i = \frac{1}{ln(i)}$

4. Prove that if $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is an alternating series ($a_n \geq 0$) and $|a_{n+1}| < |a_n|$ and $lim_{n\to\infty}a_n = 0$, the series converges.

5. According to problem 4, the alternating harmonic series, $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-...$ converges. Find its sum (hint: use $1/(1+x) = 1-x+x^2-x^3+x^4-...$).

6. Is it true that $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...$ is equal to $(1+\frac{1}{3}+\frac{1}{5}+...)-(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...)$?

7. In problem 4, if we remove the assumption $|a_{n+1}| < |a_n|$, prove that the alternating series still converges, or find a counterexample to show it may not.

8. In problem 4, if we remove the assumption $lim_{n\to\infty}a_n = 0$, prove that the alternating series still converges, or find a counterexample to show it may not.