Oct 24 Homework

1. Use square tiles of appropriate size to prove that it is possible to color the plane with 9 colors, in such a way that no two points of unit distance apart have the same color.

2. As mentioned in the text, in 1976 it was finally proved that any map can be colored using 4 or fewer colors (so that any two states with a common boundary of nonzero length have different colors). Draw a map that cannot be colored using 3 colors, thus proving that 4 is indeed the minimum required for general 2D maps.

3. A "graph" is defined as a set of vertices, and edges connecting some of the vertices. The "complete" graph K_n is a graph with n vertices, with edges connecting every pair of vertices (so K_3 for example is a triangle, basically). A "planar" graph is defined to be a graph which CAN BE drawn with no edges crossing (except at vertices, naturally).

   a. How many edges does K_n have?

   b. Show that K_2, K_3, and K_4 are planar.

   c. Show that the famous 4-color theorem implies that K_n is not planar, for $n > 4$. (Hint: assume K_n is planar, then construct a map of n states, each of which has a border with the other n-1.)

4. What is the minimum number of colors that can be used to color the 6 faces of a cube, so that no faces sharing a common edge have the same color? Explain how to color them using the minimum number of colors.

5. Consider 3D "maps", where the states are solids, and we want any two states which share a common boundary to have different colors. (Now only 2D surfaces can be considered boundaries, a point or 1D curve is not a boundary.) Show that the four-color theorem does not hold for 3D maps, by constructing a 3D map that requires at least 5 colors. (Hint: find a solid which can be sectioned into 5 parts, each of which has a boundary with each of the other 4.)

6. Stack bricks in the simplest way, where brick (i,j,k) occupies the space $i < x < i + 1, j < y < j + 1, k < z < k + 1$. What is the minimum number of colors for this 3D map (explain)?

7. Modify Schumer problem 10.6 so that up to 15 days are worked, and a 15-inch gold bar is to be cut into 4 parts. (Hint: every number from 0 to 15 has a binary representation of 4 or fewer bits.)