a. Solve the 1D steady-state beam bending problem of Exercise 4.2.8 (p84), with E = 10, I(x) = 1, L = 2.5 (E = elastic modulus, I = moment of inertia, L = beam length). Also, put f(x) = -1, so there is a constant downward force, namely the weight of the uniform beam itself. The boundary conditions specify that the beam is clamped at the left end, and free and unsupported at the right end. Also solve the boundary problem analytically and compute the L₁ norm of the error, that is, the integral of |u - u_{true}|. Set E, I, L, f as parameters, so their values could be easily changed. Note that parameters and variables must begin with a letter in the range A - H or O - Z, otherwise they will be integers, for example, L would be rounded down to 2, so use (for example) RL instead of L.

Use the PDE2D GUI (pde2d_gui [name]) which means the collocation method will be used.

- b. Repeat the problem using the Galerkin method. For this you must use the Interactive Driver (pde2d [name]). Notice that the format for the PDEs and (especially) the boundary conditions are very different than for the collocation method, so pay close attention to the documentation, especially to the hint on how to handle "mixed" (eg, one fixed, one free) boundary conditions.
- a. Solve the 1D time-dependent heat conduction/convection problem of Example 6.1 (p108), with ρ = 1.1, C = 0.18, v = 2.0, κ = 0.1, L = 5, T₀ = 30, T_{out} = 10, k/R = 0.2, T_{init}(z) = T₀ + (T_{out} T₀)z/L (ρ = fluid density, C = fluid specific heat, v = fluid velocity, κ = fluid heat conductivity, L = pipe length, T₀ = temperature of entering fluid, T_{out} = temperature of pipe wall and outlet, k = heat transfer coefficient between fluid and pipe wall, and R = pipe radius). Solve this forward in time to t = 2. Note that the spatial variable must be called x, not z, and you will not be able to call the temperature T, because T is reserved for time, so call it TMP, and use names TMP0, TMPout in place of T0,Tout. You can use EITHER the collocation OR Galerkin method.



b. Now replace the boundary condition at the outlet (z = L) by the nonlinear radiation boundary condition at the top of page 68, with $\sigma = 10^{-5}$ (σ = radiation heat transfer coefficient) and repeat the problem. You will need to reset the variable LINEAR to .FALSE. (collocation method only), and FDIFF to .TRUE..



c. Repeat the problem with no conduction, $\kappa = 0$. Now this is the hyperbolic transport problem of Example 8.1 (p153), and there should now be "no" boundary condition at the outlet, since it is first order in space. Note that without conduction (diffusion of heat), the discontinuity in the derivative is not smoothed out now, but simply transported forward with velocity v.

