POLYHEDRA MODELS

Icosahedron, Dodecahedron,
Great Stellated Dodecahedron
Icosahedron Model

The instructions for the icosahedron model follows on the next page. For the value of $X$, use $X = 5 \text{ cm}$. 
Icosahedron

Construction of the Model

1. You are given value of $X$ cm for the radius of the circumcircle about one of the equilateral triangular faces of the model.

3. Construct an equilateral triangle with side equal to the radius of the circumcircle. You need not use a protractor. The compass is more accurate.

3. The model of the icosahedron consists of 20 congruent equilateral triangles. If the model is dissected into a plane figure, it appears as the network of triangles shown below.

4. The model can be easily constructed by making 20 congruent equilateral triangles and connecting them with paper hinges and glue. Any vertex of the icosahedron is surrounded by 5 equilateral triangles. The last piece is glued as a hinged door. By applying glue to the underlying tabs, the triangular-door can be closed to align to the edges.
The instructions for the dodecahedron model follows on the next page. For the value of $X$, use $X = 5 \text{ cm}$. 
Polyhedra
MATH 2304
Dr. Swienciki

Dodecahedron $5^3$

Dual of Icosahedron

Euler’s Formula:

$F + V = E + 2$

$F: 12$ $V: 20$ $E: 30$

Edge

Inter-Radius

$= 3 - \sqrt{5}$

0.7639

Dihedral angle $= 116\degree34\prime (= \pi - \tan^{-1}2)$

Construction of the Model

1. You are given a value of $X$ cm for the radius of the circumcircle, enclosing one of the regular pentagon faces of the model.

2. Construct the regular pentagon. Use a compass, protractor and ruler. Check the 5 edges of the pentagon for accuracy before you continue.

3. The model of the dodecahedron consists of 12 congruent regular pentagons. If the model is dissected into a plane figure, it appears as the network of regular pentagons shown below.

4. The model can be easily constructed by making 12 congruent regular pentagons and connecting them with paper hinges and glue. Any vertex of the dodecahedron is surrounded by 3 regular pentagons. The last piece is glued as a hinged door. By applying glue to the underlying tabs, the pentagon-door can be closed to align to the edges.
Great Stellated Dodecahedron

The instructions for the *great stellated dodecahedron* model follows on the next page. For the dodecahedron inside the model, use the value of $e = 5\ cm$. (You construct a second model of the dodecahedron for the inside of the star.)

When you construct the 20 pyramids, which attach, make the base of the pyramid $e = 4.95\ cm$. This is done because the cardboard has thickness and the slightly smaller base will allow for the expansion and the pyramids will attach better.
The Great Stellated Dodecahedron

The great stellated dodecahedron is one of the four beautiful solids known as the Kepler-Poinsot polyhedra. These were unknown in ancient times. The great stellated dodecahedron was discovered by the mathematician Poinsot (1777-1859). This polyhedron is considered to be regular, but it is not convex as the five regular Platonic solids. Convex polyhedra have the property that between any two points on the solid, the segment between the two points is contained within the solid.

The construction of the model can best be done by constructing the regular icosahedron and attaching twenty pyramids with base an equilateral triangle and lateral faces consisting of isosceles triangles with base angles of 72°.
Construction of the great stellated dodecahedron

To Construct the polyhedron, we define the following parts of the solid:

L : The edge of the great stellated dodecahedron (This is the length of the segment between points on the star).

e : edge of the icosahedron inscribed by the great stellated dodecahedron.

This is also the edge of the base of the isosceles triangle in the figure to the right.

R : The inter-radius of the icosahedron (This is the radius of the sphere that circumscribes the icosahedron).

r : The radius of the equilateral triangle or the radius of the circle circumscribing the equilateral triangle (This refers to the equilateral triangle which forms one of the 20 faces of the equilateral triangle, forming the faces of the icosahedron, inside the polyhedron.

Value of L = _____

Calculations:

1. Make the following calculations from the value of L:
   
   R = 0.23L;
   e = 1.236 R;
   r = 0.58 e.

2. Round off answers to the nearest millimeter, after calculating r.

Steps:

1. Construct a circle of radius r;
   Construct an equilateral triangle, inscribed in the circle;
   Construct twenty equilateral from the template of the equilateral triangle and complete the icosahedron, as previously studied in geometry.

2. Construct the template above, using e as the base of one of the three congruent isosceles triangles. Construct 20 copies of the template above, adding tabs to the base of the three equilateral triangles and one of the outer legs. You glue the twenty pramids to the faces of the icosahedron.