Icosahedron $3^5$

**Construction of the Model**

1. You are given a value for the inter-radius, $r = 5$ cm.
2. Calculate the edge length from the inter-radius, and round your answer off to the nearest tenth.
3. Construct an equilateral triangle with side equal to the edge you calculated in step 2. Use a compass and straight edge only.
4. The model of the icosahedron consists of 20 congruent equilateral triangles. If the model is dissected into a plane figure, it appears as the network of triangles shown below.
5. The model can be easily constructed by making 20 congruent equilateral triangles and connecting them with paper hinges and glue. Any vertex of the icosahedron is surrounded by 5 equilateral triangles. The last piece is glued as a hinged door. By applying glue to the underlying tabs, the triangular-door can be closed to align to the edges.
Dodecahedron 5³

Dual of Icosahedron

Euler's Formula:
\[ F + V = E + 2 \]

\[ F: 12 \quad V: 20 \quad E: 30 \]

\[
\text{Edge Inter-Radius} = 3 - \sqrt{5} \approx 0.7639
\]

Dihedral angle = 116°34′ (= \pi - \tan^{-1}2)

Construction of the Model

1. You are given a value for the inter-radius, \( r = 8 \text{ cm} \).
2. Calculate the edge length from the inter-radius. Round your answer to the nearest tenth.
3. Construct a regular pentagon with side equal to the edge you calculated in step 2. Use a compass, protractor, and centimeter ruler.

Hint: Construct an isosceles triangle with base congruent to the side of the regular pentagon, and base angles equal to twice the vertex angle.

4. The model of the dodecahedron consists of 12 congruent regular pentagons. If the model is dissected into a plane figure, it appears as the network of regular pentagons shown below.

5. The model can be easily constructed by making 12 congruent regular pentagons and connecting them with paper hinges and glue. Any vertex of the dodecahedron is surrounded by 3 regular pentagons. The last piece is glued as a hinged door. By applying glue to the underlying tabs, the pentagon-door can be closed to align to the edges.
The *Stella Octangula* was named by Johannes Kepler (1571–1630), but was studied earlier by Fra Luca Bartolomeo de Pacioli (1445–1517), who called it the *octaedron elevatum*.

The solid is a compound of two intersecting tetrahedra and can be considered as a *stellation* of the octahedron.

From the figure below, you can visualize that the polyhedron an be inscribed inside of a square. The diagonals on each square face align with the edges of the tetrahedra. They also demonstrate that the tetrahedral edges intersect at right angles on each square face.

**Construction**

You are to construct the model of the stella octangula so that it can be inscribed in a square with edge $e = 20$ cm.

Round off measurements to the nearest tenth.
Use 10 cm for the edge of each equilateral triangle in the network of figure 1.

The Stella Octangula can be constructed by making the regular octahedron first. The network is shown in the figure 1. It is composed of equilateral triangles and tabs, which can be scored, folded, and glued to the underside of the faces.

On the next page, the network for the regular tetrahedra, which are glued to each of the faces of the octahedron is shown in figure 2.
Tetrahedral pyramids

You need to construct 8 tetrahedral pyramids as shown in figure 2. There are tabs illustrated in the figure, which you use to glue to each faces of the regular octahedron you have already constructed.

The edge of each tetrahedral pyramid would seem to be 10 cm. However, because of the thickness of the card stock or poster-board material, it is best to use 9.9 cm for the edge of each of the equilateral triangles in the network of figure 2.

To show the stella octangula as a compound of two large intersecting tetrahedra, make 4 tetrahedral pyramids in a contrasting color with respect to the other 4 pyramids.

Tetrahedral Pyramid Network

figure 2
Completing the Stella Octangula

Eight Pyramids

The 8 tetrahedral pyramids are shown in figure 3. Use the tabs to glue each pyramid to the octahedron, in the pattern shown on the completed model.
1. Construct the regular pentagon with radius \( r = 4 \text{ cm} \), as shown in figure 1.

2. Construct a regular hexagon with radius \( AB \), as shown in figure 2.
Truncated Icosahedron

3. Use figure 1 as a template. Punch out 12 copies on card stock, using a push pin.

4. Use figure 2 as a template. Punch out 20 copies on card stock, using a push pin, as shown in figure 4. Draw the tabs on each side of the hexagons, as shown.
5. Each pentagon is surrounded by 5 hexagons. The pattern is the same throughout the polyhedron, as shown in figure 5.

Score the edges of the hexagon, making a clean fold.

Use Elmer’s Glue All to adhere the tabs under the edges of the pentagons.
6. The last piece to glue is the regular pentagon.

You will notice the cavity in the figure with 5 tabs.

Glue one of the edges of the regular pentagon and let it dry. Then, apply the glue to the other 4 tabs and align the edges of the pentagon carefully to dry.
1. Construct the regular decagon with radius $r = 5\text{ cm}$, as shown in figure 1.

2. Construct a regular hexagon with radius $r = AB$, as shown in figure 2.

3. Construct a square with side $AB$, as shown in figure 3.
4. Use the template from figure 1 to make 12 copies of the decagon parts, shown in figure 4, using card stock.

5. Use the template from figure 2 to make 20 copies of the regular hexagon parts as shown in figure 5, using card stock.

6. Use the template from figure 3 to make 30 copies of the square shown in figure 6, using card stock.
7. In figure 7, you see how every vertex of the Rhombicosidodecahedron connects. This is the pattern of a semi-regular solid, described with the symbols or notation 4\times6\times10.

The tabs of the green hexagons can be cut off and the tab of the red dodecagon can be aligned and glued underneath it.

The squares do not need tabs. The tabs of the regular hexagon and regular decagon can be aligned and glued underneath it. The last piece to be glued to close the model is the square, as shown in figure 8, page 4.
8. In figure 8, you glue the last piece to the 4-tabs, shown in figure 8.

Once you align one square edge and glue it to a tab, it appears to work like a hinged door.

One the first tab is dried to the underside of the square, you apply glue to the other three tabs and align the edges of the square, until the glue sets.