Circles

PTOLEMY'S
INSCRIBED PENTAGON
Definitions and Notation in Circles

In Circle O, O O:

- Inscribed Angle: \( \angle FGH \)
- Central Angle \( \angle AOC \)
- Arc ABC or Arc AD or Arc FH
- Chord AB or GH or GF
- Diameter ED

Properties: The Central Angle is measured by its intercepted arc.

The Inscribed angle is measured by one-half its intercepted arc.

If an arc is less than 180°, it is called a minor arc; and if greater it is called a major arc.

Minor arc + Major arc = 360°
Definitions and Properties in Circles

In Circle O, OO:

In the figure at the right, the secant and the tangent are straight lines. They are related, but not the same concept as the tangent and secant functions found in the subject of trigonometry.

Any diameter is of a circle is also a chord. It is the longest chord.

A circle is the set of points in a plane drawn equidistant from a fixed point called the center.

The radius of a circle is any segment drawn from the center to any point on the circle.

The tangent is a line drawn in the plane a circle with exactly one point in common with the points on the circle. The common points is called the point of tangency.

A semicircle is the set of points on a circle consisting of the end points of a diameter and all points on one side of the circle.

Properties:

If a radius is drawn to the point of tangency, the tangent and radius are perpendicular.

If a radius is drawn perpendicular to a chord, it bisects the chord and its intercepted arc.

An angle inscribed in a semicircle is a right angle.
Sample Applications

1. In Circle O, AD is a diameter and BD = 80°.

Find:

\[ \angle DEB = \, ___ \]
\[ \angle DAB = \, ___ \]
\[ \angle ADB = \, ___ \]
\[ \angle ABD = \, ___ \]
\[ \angle DOB = \, ___ \]
\[ \angle AOE = \, ___ \]
\[ \angle AOD = \, ___ \]
\[ \angle BOE = \, ___ \]
\[ \angle ADE = \, ___ \]
\[ \angle ABE = \, ___ \]

2. In Circle O, AB is a diameter and \( \angle CAO = 35° \)

Find:

\[ \angle CBA = \, ___ \]
\[ \angle ACB = \, ___ \]
\[ \angle COB = \, ___ \]
\[ \angle OCB = \, ___ \]
\[ \angle AOC = \, ___ \]
\[ \angle ACO = \, ___ \]
Sample Applications

3. In Circle O, \( \overline{AB} \), \( \overline{CD} \) are chords, intersecting at E.
\[ \angle CAB = 40^\circ \]
\[ \overarc{ARD} = 200^\circ \]
Find:
\[ \angle CDB = _____ \]
\[ \angle ACD = _____ \]
\[ \angle ABD = _____ \]
\[ \angle AEC = _____ \]
\[ \angle BED = _____ \]
\[ \angle CEB = _____ \]
\[ \angle AED = _____ \]

4. In Circle O, radius AO = 10 units.
There is a chord \( \overline{AB} \) located 6 units from the center.
Find: length of chord \( \overline{AB} \)
Sample Applications

Theorem 1. Tangent segments to a circle drawn from the same point are congruent/equal.

Given or Hypothesis:
In Circle $O$, $AB$ and $CD$ are tangents intersecting inside the circle.
Conclusion:

Theorem 2. If two chords or secants intersect within a circle, the vertical angles formed equal one-half the sum of the intercepted arcs.

Given or Hypothesis:
In Circle $O$, $AB$ and $CD$ are secant lines intersecting inside the circle.
Conclusion:
\[
\angle BED \text{ or } \angle CEA = \frac{1}{2} ( \overarc{BVD} + \overarc{AWC} )
\]
Math 3304
Circles
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Sample Applications

Theorem 3. The opposite angles of a quadrilateral inscribed in a circle are supplementary.
(Sometimes this is called a cyclic quadrilateral.)

Given or Hypothesis:
In Circle O, A, B, C, D are points on the circle and
ABCD is a quadrilateral.

Conclusion:
\( \angle A \) and \( \angle C \) are supplementary and
\( \angle B \) and \( \angle D \) are supplementary

Theorem 4. If two secants intersect in the exterior of their circle, the angle formed equals one-half the
difference of their intercepted arcs.

Given or Hypothesis:
In Circle O, \( \overline{AB} \) and \( \overline{CD} \)
are secant lines intersecting outside the circle.

Conclusion:
\[ \angle BED = \frac{1}{2}(\angle BEC - \angle BDE) \]
Sample Applications

Theorem 5. The angle formed by a secant and tangent drawn form a point on the circle equals one-half their intercepted arc.

Given or Hypothesis:
In Circle O, BD is a tangent and BA is a secant drawn from point B on the circle.

Conclusion:
\[ \angle ABD = \frac{1}{2} (\widehat{ACB}) \]

Theorem 6. The angle formed by a secant and tangent intersecting in the exterior of their circle equals one-half the difference of their intercepted arcs.

Given or Hypothesis:
In Circle O, AE is a tangent and EC is a secant drawn from point E outside the circle.

Conclusion:
\[ \angle AED = \frac{1}{2} (\widehat{AWC} - \widehat{AVD}) \]
Sample Applications

**Theorem 7.** The angle formed by two tangents intersecting in the exterior of their circle equals one-half the difference of their intercepted arcs.

**Given or Hypothesis:**

In Circle O, AE and CE are tangents drawn from point E exterior to the circle.

**Conclusion:**

\[ \angle AEC = \frac{1}{2} ( \overarc{AWC} - \overarc{AVC} ) \]

**Theorem 8.** If two chords intersect within a circle, the product of the segments on one chord equals the product of the segments on the other chord.

**Given or Hypothesis:**

In Circle O, AB and CD are secants intersecting at point V in the interior of the circle.

**Conclusion:**  

\[ AV \cdot VB = DV \cdot VC \]