Factoring Polynomials – Intermediate Algebra

Fact – If a polynomial cannot be factored it is said to be prime.

Fact – The greatest common factor (GCF) is the largest common factor shared in common with all the terms of the polynomial.

Examples: Factor out the GCF.

$6a^2 = 2\cdot 3\cdot a\cdot a \to 3a$

$10a = 2\cdot 5\cdot a \to 5$

GCF = 2a

1. $6a^2 + 10a$

$6a^2 + 10a = 2a(3a + 5)$

2. $8x^3y^2 - 7x^2y$

GCF = x^2y

$8x^3y^2 - 7x^2y = x^2y(8xy - 7)$

3. $10m^2 + 4m - 2$

GCF = 2

$10m^2 + 4m - 2 = 2(5m^2 + 2m - 1)$

4. $3y(y + 4) - 5(y + 4)$

GCF = y+4

$3y(y+4) - 5(y+4) = (y+4)(3y - 5)$

Factor by Grouping – If a polynomial has four terms, try these steps:

- Factor out the GCF
- Group the first two terms and the last two terms.
- Factor out the GCF for the first two terms.
- Factor out the GCF for the last two terms.
- Factor out the GCF of the remaining expression.

Examples: Factor by grouping.

1. $6p^2 - 10p + 15p - 25$

$2p(3p-5) + 5(3p-5)$

$\boxed{(3p-5)(2p+5)}$

2. $8m^2 + 12mn - 10mn - 15n^2$

$4m(2m+3n) - 5n(2m+3n)$

$(2m+3n)(4m-5n)$

Factoring Quadratics of the form $x^2 + bx + c$.

m	n
c	b

- Find two numbers, *m* and *n*, that multiply to give *c* and add to give *b*.
- The quadratic will factor as $(x+m)(x+n)$
- Hints: The sign of *c* will tell you if the two numbers are the same sign or opposite signs; the sign of *b* tells you
  - The sign of the numbers if they are the same (*c* is positive).
  - The sign of the bigger number if they are different (*c* is negative).

Examples: Factor the following.

1. $x^2 + 9x + 14$

same

14 m	a
1·14	15
2·7	9

m×n

$x^2 + 9x + 14 = (x+2)(x+7)$

.

15	8
1 15	
3 5	8

Bonus $x^2+5x-6 = (x+6)(x-1)$

-6	+5
+6 -1	5
-2 3	

2. $x^2 + 8x + 15$

$(x+5)(x+3)$

3. $m^2 + 3m + 24$

prime

24	3
1 24	25
2 12	14
3 8	11
4 6	10

Bonus $x^2 - x - 12 = (x+3)(x-4)$

1 3 2

-12	-1
1 -12	
2 -6	
3 -4	-1

AC Method of Factoring Quadratics – The standard form of a quadratic is ax^2+bx+c.

- Factor out the GCF.
- Multiply *a* and *c* together.
- Find factors of *ac* that sum to *b*.
- Rewrite the middle (*bx*) term using the factors from step 3.
- Group and factor out what is in common.

Examples: Factor the following.

$ac = 15(-28) = -420$

-420	+23
-1 420	
-10 42	32
-15 28	13
-20 21	1
-12 35	23

1. $15x^2 + 23x - 28$

$15x^2 - 12x + 35x - 28$

$3x(5x-4) + 7(5x-4)$

$\boxed{(5x-4)(3x+7)}$

2. $6x^2 + 11xy + 3y^2$

$6(3y^2) = 18y^2$

$18y^2$	$11y$
1y 18y	
2y 9y	11y

$6x^2 + 2xy + 9xy + 3y^2$

$2x(3x+y) + 3y(3x+y)$

$(3x+y)(2x+3y)$

3. $6x^2 + x - 35 \longrightarrow 6x^2 - 14x + 15x - 35$

$6(-35) = -210$

-210	$+1$
$-6 \quad 35$	29
$-10 \quad 21$	11
$-15 \quad 14$	-1
$-14 \quad 15$	$+1$

$2x(3x-7) + 5(3x-7)$

$(3x-7)(2x+5)$

4. $4x^2 - 7x + 3 \longrightarrow 4x^2 - 3x - 4x + 3$

$4(3) = 12$

12	-7
$-3 \quad -4$	-7

$x(4x-3) - 1(4x-3)$

$(4x-3)(x-1)$

$3(4) \quad 3(29) \quad 3(7)$

5. $12x^3 - 87x^2 + 21x = 3x(4x^2 - 29x + 7)$

$GCF = 3x$

$3x(4x^2 - 1x - 28x + 7)$

$3x(x(4x-1) - 7(4x-1))$

$3x(4x-1)(x-7)$

$4(7) = 28$

M	A
28	-29
$-1 \quad -28$	