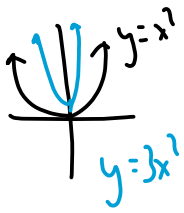
Graphing Quadratics in Vertex Form – Intermediate Algebra

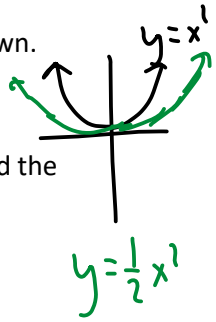
Definition – The vertex of a quadratic equation in vertex form can be read directly from the equation. That is, $f(x) = a(x-h)^2 + k$ has vertex (h, k).

Note: In this form, the *h* is being subtracted from *x*, therefore the *x*-coordinate of the vertex has the opposite sign than the one you see in the formula.

Facts – The constants *a, h,* and *k* all tell you something about the graph of *f*.



a. If $a > 0$ the parabola opens up. (Positive people smile.) If $a < 0$ the parabola opens down. (Negative people frown.)
b. Ignoring the sign, if $a > 1$ the graph is narrower and if $a < 1$ the graph is wider.
c. The value of *h* tells us if the graph shifts left or right: if $h > 0$ then we have $(x-h)$, and the graph shifts right; if $h < 0$ then we have $(x+h)$, and the graph shifts left.
d. The value of *k* tells us if the graph shifts up or down:
   if $k > 0$ the graph shifts up and
   if $k < 0$ the graph shifts down.



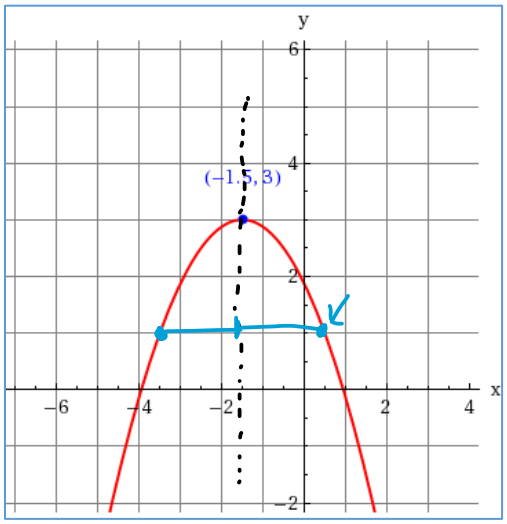
Fact – The domain of EVERY quadratic function consists of all real numbers, $(-\infty, \infty)$. The range of a quadratic depends on the vertex, (h, k), and the leading coefficient *a*.

If $a > 0$ the graph opens up and the range is $y \geq k$ or $[k, \infty)$.

If $a < 0$ the graph opens down and the range is $y \leq k$ or $(-\infty, k]$.

Examples:

1. Use the graph to answer the questions.



a. Which point is the vertex?

(-1.5, 3)

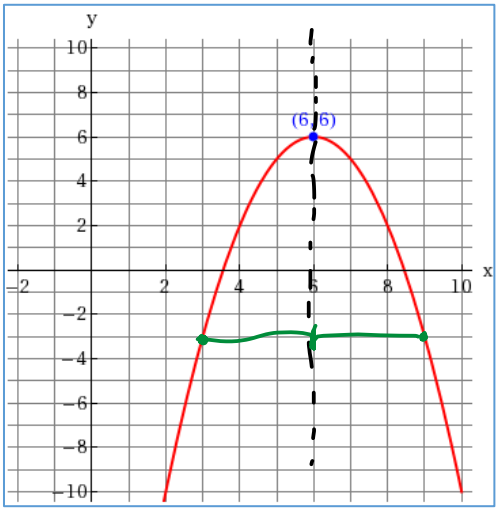
b. What is the equation of the axis of symmetry?

$x = -1.5$

c. What is the symmetric point to the point $(-3.5, 1)$?

(0.5, 1)

2. Use the graph to answer the questions.



a. Which point is the vertex?

(6,6)

b. What is the equation of the axis of symmetry? $x = 6$

c. Would the value of *a* be positive or negative? How do you know?

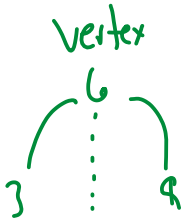
negative a as graph opens down

d. In vertex form, what are the values of *h* and *k*? $h = 6, k = 6$

$f(x) = a(x-6)^2 + 6$

e. What is the symmetric point to the point (9,-3)?

(3,-3)



To Graph a Quadratic Function from the Vertex Form:

1. Determine whether the graph opens up or down.
2. Find the vertex and the equation for the axis of symmetry.
3. Find the vertical intercept: $f(0)$
4. Find an extra point by doubling the *x*-coordinate of the vertex and using the *y*-coordinate of the *y*-intercept: $(2h, f(0))$.
5. Plot these points and any extras as needed.

3. Sketch the graph of the given functions. Label the vertex, vertical intercept, and at least one additional symmetric point. State the domain and range of each function.

a. $f(x) = (x-4)^2 + 3$

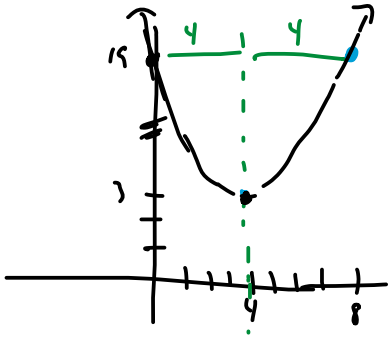
$a = 1$ (pos) $\cup$

vertex $(4, 3)$

A.O.S. $x - 4 = 0$

$x = 4$

$f(0) = (0-4)^2 + 3 = 19$



$h = 4$

$2h = 8$

$(8, 19)$

D: $(-\infty, \infty)$

R: $[3, \infty)$

b. $g(x) = (x+2)^2 - 1$

$a = 1$ up

A.O.S. $x + 2 = 0$

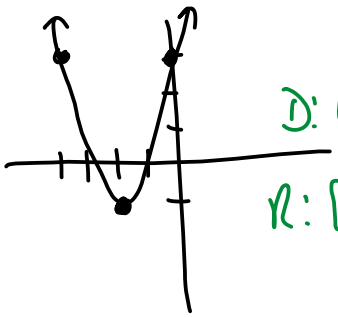
$x = -2$

vertex $(-2, -1)$

$f(0) = (0+2)^2 - 1 = 3$

$(0, 3)$ vert int.

$(-4, 3)$ sym pt



D: $(-\infty, \infty)$

R: $[-1, \infty)$

c. $h(x) = -2x^2 + 5 = -2(x-0)^2 + 5$

$a = -2$ down

$x - 0 = 0$

$x = 0$ axis of sym.

$(0, 5)$ vertex

y-int

$f(0) = -2(0)^2 + 5 = 5$

$f(1) = -2(1)^2 + 5 = 3$

$f(-1) = 3$