

Solving Quadratic Equations by the Square Root Property

Square Root Property – If $c \geq 0$, the solutions to the equation $x^2 = c$ are $x = \pm\sqrt{c}$. If c is negative, then the equation has no real solutions.

Steps to Solving Quadratic Equations Using the Square Root Property

1. Isolate the squared variable expression
2. Use the square root property to undo the square. Use \pm on the side away from the variable.
3. Rewrite as two equations and solve.
4. Check answer(s) in the original equation. Be sure that answers are within the domain of the model and that they make sense in the context of the problem.

Examples: Solve

1. $x^2 - 4 = 0$

$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$x = \sqrt{4}$$
$$x = 2$$

$$x = -\sqrt{4}$$
$$x = -2$$

2. $x^2 + 9 = 0$

$$x^2 = -9$$

No real solutions

3. $-3x^2 + 12 = -15$

$$\frac{-3x^2 + 12}{-12} = \frac{-15}{-12}$$

$$-3x^2 = -27$$

$$\frac{-3x^2}{-3} = \frac{-27}{-3}$$

$$x^2 = 9$$

$$x^2 = 9$$
$$x = \pm\sqrt{9}$$

$$x = \sqrt{9}$$
$$x = 3$$

$$\text{or } x = -\sqrt{9}$$
$$\text{or } x = -3$$

4. $(p - 5)^2 - 20 = 4$

$$\frac{(p - 5)^2 - 20}{+20} = \frac{4}{+20}$$

$$(p - 5)^2 = 24$$

$$p - 5 = \pm\sqrt{24}$$

$$\left\{ \begin{array}{l} \sqrt{24} = \sqrt{4 \cdot 6} \\ = 2\sqrt{6} \end{array} \right.$$

$$p - 5 = 2\sqrt{6} \rightarrow p = 5 + 2\sqrt{6}$$

$$p - 5 = -2\sqrt{6} \rightarrow p = 5 - 2\sqrt{6}$$

$$1.5y - 14 = 82$$

$$5. \quad 1.5(n-4)^2 - 14 = 82$$

$$\begin{array}{r} +14 \quad +14 \\ \hline 1.5(n-4)^2 = 96 \\ (n-4)^2 = \frac{96}{1.5} \\ (n-4)^2 = 64 \end{array}$$

$$\rightarrow n-4 = \pm\sqrt{64}$$

$$n-4 = \sqrt{64}$$

$$n-4 = 8$$

$$\begin{array}{r} +4 \quad +4 \\ \hline n = 12 \end{array}$$

$$\text{or } n-4 = -\sqrt{64}$$

$$n-4 = -8$$

$$\begin{array}{r} +4 \quad +4 \\ \hline n = -4 \end{array}$$

$$6. \quad 3 = 5(x+3)^2 - 17$$

$$\begin{array}{r} +17 \quad +17 \\ \hline 20 = 5(x+3)^2 \\ \frac{20}{5} = \frac{5(x+3)^2}{5} \end{array}$$

$$4 = (x+3)^2$$

$$x+3 = \pm\sqrt{4}$$

$$x+3 = \sqrt{4}$$

$$x+3 = 2$$

$$\begin{array}{r} -3 \quad -3 \\ \hline x = -1 \end{array}$$

$$\text{or } x+3 = -\sqrt{4}$$

$$\text{or } x+3 = -2$$

$$\begin{array}{r} -3 \quad -3 \\ \hline x = -5 \end{array}$$