Finding Equations of Lines – Intermediate Algebra

✗ Steps to find the equation of a line using the slope-intercept form $y = mx + b$:

✓ 1. Use any two points to calculate the slope.
✓ 2. Substitute in the slope and a point to find the value of b.
✓ 3. Write the equation in slope-intercept form.
✓ 4. Check the equation by plugging in the points to make sure they are solutions.

We can also use the point-slope form of a line: $y - y_1 = m(x - x_1)$ to find the equation. Using this form, we will always simplify the answer into slope-intercept form.

✳ Steps to find the equation of a line using the point-slope formula $y - y_1 = m(x - x_1)$:

✓ 1. Use any two points to calculate the slope.
✓ 2. Substitute in the slope and a point into the point-slope formula.
✓ 3. Write the equation in slope-intercept form.

Check the equation by plugging in the points to make sure they are solutions.

Examples: Write the equation of the line that passes through the given points. Use both methods so that you can get a feel for each one.

1. (4,3) and (20, −17)

① $m = \dfrac{-17-3}{20-4} = \dfrac{-20}{16} = \dfrac{-4(5)}{4(4)} = \dfrac{-5}{4}$

$\boxed{y = -\dfrac{5}{4}x + 8}$

$y = mx + b$

$3 = -\dfrac{5}{4}(4) + b$

$3 = -5 + b$

$8 = b$

$-17 = -\dfrac{5}{4}(20) + 8$

$-17 = -25 + 8$ ✓

2. (3,7) and (8,17)

$m = \dfrac{17-7}{8-3} = \dfrac{10}{5} = 2$

$y - y_1 = m(x - x_1)$

$y - 7 = 2(x - 3)$

$y - 7 = 2x - 6$

$+7 \qquad +7$

$\boxed{y = 2x + 1}$

$17 = 2(8) + 1$

$17 = 16 + 1$ ✓

$$y - y_1 = m(x - x_1)$$

3. $(-2, -3)$ and $(5, 9)$

$$m = \frac{9 - (+3)}{5 + (+2)} = \frac{12}{7}$$

$$y - 9 = \frac{12}{7}\left(x - 5\right)$$

$$y - 9 = \frac{12}{7}x - \frac{60}{7}$$
$$\quad +9 \qquad\quad +9$$
$$\boxed{y = \frac{12}{7}x + \frac{3}{7}}$$

$$-\frac{60}{7} + \frac{9\cdot7}{1\cdot7}$$

$$-\frac{60}{7} + \frac{63}{7} = \frac{3}{7}$$

Examples: Find an equation for a linear model.

1. The number of sports cards hobby stores has been declining steadily. In 1995, there were 4500 stores, and in 2005 there were only 1500. Write an equation for the line that gives the number of sports card hobby stores given the number of years since 1995.

let $x = $ # of yrs since 1995

$(0, 4500) \quad (10, 1500)$

$$m = \frac{1500 - 4500}{10 - 0} = \frac{-3000}{10} = -300$$

$$y - 4500 = -300(x - 0)$$
$$y - 4500 = -300x$$
$$y = -300x + 4500$$

$$\boxed{y = -300x + 4500}$$

2. A business purchased a production machine in 2005 for $185,000. For tax purposes, the value of the machine in 2011 was $129,500. If the business is using straight line depreciation, write the equation of the line that gives the value of the machine based on the age of the machine in years.

$x = $ age of machine

$\begin{matrix} 2011 \\ -2005 \\ \hline 6\, yr \end{matrix}$

$2005 = $ new $\Rightarrow x = 0$

$(0, 185000) \quad (6, 129500)$

$$m = \frac{129500 - 185000}{6 - 0} = -55,500$$

$$v(x) = -55,500x + 185,000$$

$$M_1 = M_2$$

Parallel Lines – Two different lines are parallel if their slopes are equal. Parallel lines will never touch.

Perpendicular Lines – Two lines are perpendicular if their slopes are opposite reciprocals. That is, if the product of their slopes is -1.

$$M_1 \cdot M_2 = -1 \longrightarrow M_1 = -\frac{1}{M_2}$$

Examples: Write the equation of a line that goes through

1. $(-12, 8)$ and is perpendicular to the line $y = 4x - 23$

$$m = -\frac{1}{4} \qquad m = \frac{4}{1}$$

$$y - 8 = -\frac{1}{4}(x + (+12)) \longrightarrow y - 8 = -\frac{1}{4}x - 3$$
$$ + 8 \phantom{= -\frac{1}{4}x} + 8$$

$$\boxed{y = -\frac{1}{4}x + 5}$$

2. $(8, 11)$ and is parallel to the line $5x - 2y = 30$

use
$$m = \frac{5}{2}$$

find m
$$\frac{-2y}{-2} = \frac{-5x}{-2} + \frac{30}{-2}$$
$$y = \boxed{\frac{5}{2}}x - 15$$

$$y - 11 = \frac{5}{2}(x - 8)$$
$$y - 11 = \frac{5}{2}x - 20$$
$$ + 11 \phantom{= \frac{5}{2}x} + 11$$
$$\boxed{y = \frac{5}{2}x - 9}$$

3. $(-9, 3)$ and is perpendicular to the line $3x + 7y = -21$

$$m = +\frac{7}{3}$$

find m
$$\frac{7y}{7} = \frac{-3x}{7} - \frac{21}{7}$$
$$y = \boxed{-\frac{3}{7}}x - 3$$

$$y - 3 = \frac{7}{3}(x + (+9))$$

$$y - 3 = \frac{7}{3}x + \frac{63}{3}$$

$$y - 3 = \frac{7}{3}x + 21$$
$$ + 3 \phantom{= \frac{7}{3}x} + 3$$

$$\boxed{y = \frac{7}{3}x + 24}$$