Functions and Function Notation

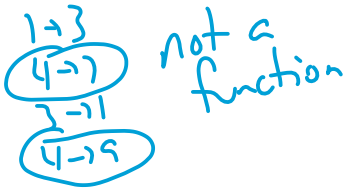
**Relation** – A relation is a set of ordered pairs.
**Function** – A relation in which each input is related to only one output. For each input value in the domain, you must have one and only one output value in the range.

Examples: Determine whether the following descriptions of relations are functions or not.

1. The set $A = \{(2,5), (4,8), (10,8), (20,15)\}$

input 2→5, 4→8, 10→8, 20→15 function

2. The set $B = \{(1,3), (4,7), (3,1), (4,9)\}$




not a function

3.

| Days of Week | Monday | Wednesday | Saturday | Monday |
|---|---|---|---|---|
| Temperature | 90 | 88 | 91 | 93 |

Monday ruins it! not a function

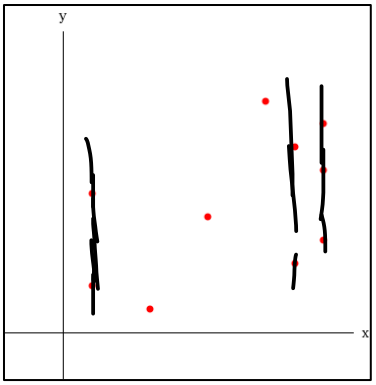
4. The population of New Mexico each year.

output input function

**Vertical line test for a function** – If any vertical line intersects a graph in at most one point, the graph represents a function.

Examples: Determine whether the following graphs are graphs of functions.

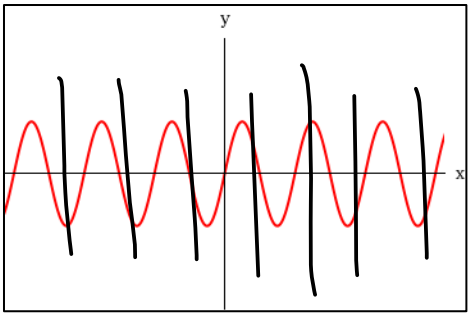
1. .



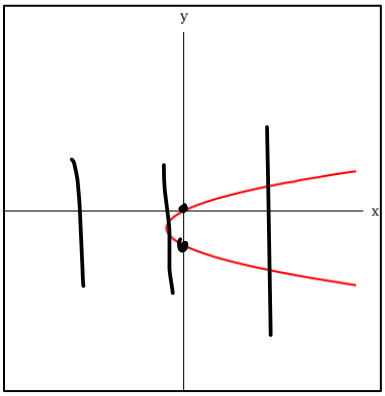

not a function

2.

function

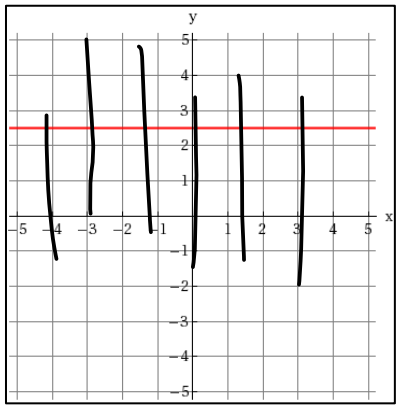



3. .




not a function

4.




function

In general, mathematicians have shortened ways of doing many things. One of these short-cuts is in how we write functions. Rather than saying *y* is a function of *x*, we would write *y = f(x)*. Similarly, we could use function notation for all of our model problems.

Examples: Specify the input and output variables and their definition and units. Determine whether or not each relation is a function.

1. $\bar{G}(a)$ = Grade level of students when they are *a* years old.

input: a = yrs of age  G: grade level at a yrs  Not a function

2. $S(a)$ = Salary, in dollars, of a person who is *a* years old.

input a = age in yrs
output: S = salary in $

not a function

3. $P(w)$ = Postage, in dollars, it takes to mail a first-class package weighing *w* ounces.

input: w = ounces (weight)

output = P = postage $

is a function

**Domain of a function** – The set of all real numbers that make the function defined is the domain of a function. Avoid division by zero and negatives under a square root.

**Range of a function** – The set of all possible output values resulting from all the values of the domain is the range of a function. A graph is very helpful in determining the range of a given function.

Examples: Let $f(x)=7x+2$, $g(x)=-1.25x+14$ and $h(x)=2x^2-10$. Find the following.

$f(\ )=7(\ )+2$ $g(\ )=-1.25(\ )+14$ $h(\ )=2(\ )^2-10$

1. $f(3)=23$

$f(3)=7(3)+2$
$=21+2$
$=23$

2. $h(-4)=22$

$h(-4)=2(-4)^2-10$
$=2(16)-10$
$=32-10=22$

3. All x such that $g(x)=15$

$-1.25x+14=15$
$-14\quad -14$
$-1.25x=1$

$\frac{-1.25x}{-1.25}=\frac{1}{-1.25}$
$x=-.8$

4. The domain and range of each function.

f linear
D: $(-\infty,\infty)$
R: $(-\infty,\infty)$

g linear
D: $(-\infty,\infty)$
R: $(-\infty,\infty)$

h quad
D: $(-\infty,\infty)$
R: $[-10,\infty)$

$2x^2-10$