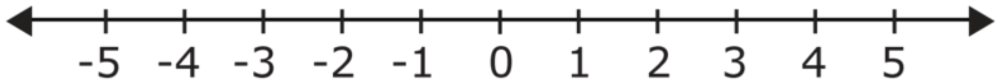
Solving Linear Inequalities – Intermediate Algebra

Solving linear inequalities is similar to solving linear equations with one major change: When you multiply or divide by a negative value, the direction of the inequality must flip. That's it, the entire secret. Why is that? Consider a number line:



$-5 < -2$ $\qquad$ $2 < 5$

Notice how the bigger number 5 is to the right of the number 2? But when you talk about negatives, the number that looks bigger, -5 is actually smaller than and hence left of -2. When we multiply or divide by a negative, the inequality changes direction to account for this.

Examples: Solve the inequalities

1. $-6b + 10 > 40$

$$\begin{aligned} -6b + 10 &> 40 \\ -10 \quad & \quad -10 \\ \hline -6b &> 30 \\ \overline{-6} \quad & \quad \overline{-6} \\ b &< -5 \end{aligned}$$

ineq.

interval $(-\infty, -5)$

2. $8g - 12 \leq 3g + 2$

$$\begin{aligned} -3g + 12 \quad & \quad -3g + 12 \\ \hline \frac{5g}{5} &\leq \frac{14}{5} \end{aligned}$$

$g \leq \frac{14}{5}$ ineq

interval $\left(-\infty, \frac{14}{5}\right]$

3. $-3x + 8 > x - 4$

$$\begin{aligned} -x \quad & \quad -x \\ \hline -4x + 8 &> -4 \\ -8 \quad & \quad -8 \\ \hline -4x &> -12 \\ \overline{-4} \quad & \quad \overline{-4} \\ x &< 3 \end{aligned}$$

$(-\infty, 3)$

4. $1.25x - 2 \geq 2x - 5$

$$-2x \qquad -2x$$

$$-0.75x - 2 \geq -5$$

$$+2 \qquad +2$$

$$-0.75x \geq -3$$

$$\ominus 0.75 \quad \ominus 0.75$$

$$x \leq 4$$

$(-\infty, 4)$

5. $-\frac{2g}{9} + 12 > 4$

$$-12 \quad -12$$

$$\left(-\frac{9}{2}\right)-\frac{2g}{9} > -\frac{8}{1}\left(-\frac{9}{2}\right)$$

$$g < \frac{72}{2} \rightarrow g < 36$$

$(-\infty, 36)$

6. $\frac{2}{3}(P + 4) < -\frac{5}{7}(2P - 12)$

$$\frac{2}{3}p + \frac{2}{3}\left(\frac{4}{1}\right) < -\frac{5}{7}(2p) - \left(-\frac{5}{7}\right)(12)$$

$$\frac{2}{3}p + \frac{8}{3} < -\frac{10p}{7} + \frac{60}{7}$$

LCD $= 3 \cdot 7 = 21$

$$\cancel{21}^{7}\left(\frac{2}{\cancel{3}}p\right) + \cancel{21}^{7}\left(\frac{8}{\cancel{3}}\right) < \cancel{21}^{3}\left(-\frac{10p}{\cancel{7}}\right) + \cancel{21}^{3}\left(\frac{60}{\cancel{7}}\right)$$

$$7(2p) + 7(8) < 3(-10p) + 180$$

$$14p + 56 < -30p + 180$$

$$+30p \quad -56 \qquad +30p \quad -56$$

$$44p < 124$$

$$\frac{44p}{44} < \frac{124}{44}$$

$$p < \frac{31}{11}$$

$\left(-\infty, \frac{31}{11}\right)$