Factoring Practice 1

(Factoring numbers)

Factoring is one of those things that you either “get” or not. If you do know how to factor numbers, skip to Factoring Practice 2. This handout is for people that just do not seem to understand factoring no matter how much they try.

Let’s start with a definition of the word factor. Factors generally come in pairs as we will think of them here. (You can also speak of prime factors which are no longer paired, but that is another topic altogether.) Factors of a particular number, say *N*, are two numbers that multiply together to give a result of *N*. If we start with the number 12, one pair of factors is 3 and 4. This is because 3x4 = 12. There are other factors of 12 as well; we know 1x12 = 12, 2x6=12, and 3x4=12. This means that 1, 2, 3, 4, 6 and 12 are factors of the number 12. Factors that give 16 include 1x16 = 16, 2x8=16, and 4x4=16. Notice that the 4 is paired with itself so when we list the factors of 16 we will have 1, 2, 4, 8, and 16. The key to factoring polynomials (Factoring Practice 2) lies in knowing how to factors numbers. As with most of mathematics, this takes practice. A little guided practice is nice though, so take a look at some factoring “cheats” that help us determine factors.

The following rules of divisibility were taken from *The Math Teacher’s Book of Lists*, by Muschla and Muschla, 1995.

For a number to have a factor of

2, it must be an even number, ending in 2, 4, 6, 8 or 0.
3, the sum of the digits of the number must have a factor of 3.
4, the number must be even and the last two digits of the number must have a factor of 4.
5, the number must end in 5 or 0.
6, it must be even and have a factor of 3
8, the number formed by the last three digits of the number have 8 as a factor.
9, the sum of the digits must have 9 as a factor.
10, the number must end in 0.
12, the number must have both 3 and 4 as factors.

Other rules are also available, but are not as straightforward as those listed. Another tip to keep in mind is that if you list all factors in pairs, once you start to hit repeats you know you are finished. That is, once I hit 3x4=12 I knew I was finished as the next number to try, 4, was already a listed factor.

Let’s try a couple more examples before you do some on your own.

Example: Find all the factor pairs of a) 28, b) 72, and c) 135

Solution: a) Let’s start with the number 28. I know 1x28=28. Since 28 is an even number, ending in an 8, 2 is a factor of 28. To find it’s pair I divide 28 by 2 to get 2x14=28. To check for three as a factor I add the digits 2 + 8 = 10. Since 10 does not have a factor of 3, neither does 28. The rule for 4 isn’t very helpful here as the number itself is only two digits. However when we checked 2, it’s pair was also an even number. This tells us that 4 is a factor of 28 and it happens that 4x7=28. The number 28 doesn’t end in 0 or 5 so 5 isn’t a factor. Since 3 is not a factor of 28, it is impossible for 6 to be a factor. Once I hit 7 I am repeating a value so I know I’m finished. The factors of 28 are 1, 2, 4, 7, 14, and 28.

b) I will try the same for 72. First, 1x72=72, next I notice that it is an even number so 2x36=72. When I add 7 + 2 = 9 this has three as a factor so 72 must also. Using division I find that 3x24=72. The factor pair with 2 was even so again 4 is a factor with 4x18=72. Five is not a factor but six is, as both two and three are factors. Using division I find 6x12=72. I try 7 using a calculator and find that it is not a factor (I also know that 7x10=70 so 72 couldn’t possibly have 7 as a factor). The trick for 8 isn’t very useful for smaller numbers either, but since the factor pair of 4 is also an even number I know that 8 is also a factor with 8x9=72. I don’t have to check 9 as it leads to a repeat and I know I have all my factors. The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

c) With fewer words, let’s attack 135: 1x135=135. It is not an even number (so 2, 4, 6, 8, … etc. are out of the question). If I add the digits 1 + 3 + 5 = 9 so three is a factor with 3x45=135. The number ends in 5 so 5x27=135. I try to divide by 7 and find that I have a remainder of two so it doesn’t go in evenly. The sum of the digits is 9 so 135 also has 9 as a factor with 9x15=135. I try 11 and 13, skipping the even numbers for the reason stated above, and end up back at 15. I have all the factors of 135 which are 1, 3, 5, 9, 15, 27, 45, and 135.

Now it’s your turn. Once you have a pretty good handle on this, go to Factoring Practice 2 to learn how to factor polynomials.

Practice Problems: Find all factors of each number.

1. 30	2. 60	3. 70
4. 105	5. 42	6. 126
7. 45	8. 56	9. 168

Practice Problem Solutions:

1. 30 has factors 1, 2, 3, 5, 6, 10, 15, and 30
2. 60 has factors 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60
3. 70 has factors 1, 2, 5, 7, 10, 14, 35, and 70
4. 105 has factors 1, 3, 5, 7, 15, 21, 35, and 105
5. 42 has factors 1, 2, 3, 6, 7, 14, 21, and 42
6. 126 has factors 1, 2, 3, 7, 9, 14, 18, 42, 63, and 126
7. 45 has factors 1, 3, 5, 9, 15, and 45
8. 56 has factors 1, 2, 4, 7, 8, 14, 28, and 56.