Factoring Practice 2

(Factoring Polynomials)

Factoring polynomials is another special skill. Some students feel like they will never get it, while others can just call out numbers and be correct every time. If you are the type that is pretty comfortable with factoring, skip to the bottom and try some of the practice problems. For those of you that struggle every semester with factoring, this handout was created with you in mind.

There are multitudes of ways to approach factoring. Some methods will be for special circumstances and other methods are for a more general approach. Rather than discussing several methods, as you have probably seen in your past classes, I'm going to only discuss one method for factoring trinomials (polynomials with three terms). In order to completely discuss trinomials, I will first talk about the greatest common factor and factoring by grouping.

Greatest Common Factor (GCF) – Every pair of numbers, or terms of a polynomial, has what is referred to as the greatest common factor. The GCF is the largest factor that is common to both (or all if there are more than two) numbers or terms. Finding the GCF of numbers relies upon your ability to find factors of numbers. If you need a review of this, please see Factoring Practice 1. The ability to find the GCF of terms of a polynomial relies on your ability to find factors of both numbers and variables. To find the GCF for variable terms of a polynomial, find any variables that every term in consideration has in common. The GCF is that variable raised to the smallest power that they share. For example, in $3x^2y^4m^2n+7xym^2n^3$, both terms have all four variables. This means that all four variables will appear in the GCF. The smallest power of *x* in common to both terms is the first power, for *y* it is the first power, for *m* it is the second power and for *n* it is also the first power. This means that our GCF is xym^2n . When we factor out a common factor we are rewriting the expression as a multiplication problem. The original expression is the "answer" of the multiplication and the GCF is multiplied by the factor that remains. For this example we have, $3x^2y^4m^2n+7xym^2n^3=xym^2n\left(3xy^3+7n^2\right)$, where the portion in parenthesis is the factor that remains.

Fact: Always look for a GCF as your first step to factoring any polynomial.

Factoring By Grouping – The method of factoring by grouping is useful when your polynomial has four terms. All you do for this method is to factor out GCF using special grouping. Let's look at the polynomial $2ax+2bx+3a+3b$. If we focus on just the first two terms, $2ax+2bx$, we see that they have a GCF of $2x$. If we focus on just the last two terms, $3a+3b$, we see that they have a GCF of *3*. Let's rewrite what we just found:

$2ax+2bx+3a+3b=2x(a+b)+3(a+b)$. This completes Step 1: Find the GCF of the first pair of terms and find the GCF of the last pair of terms and factor them out. If we look at the form of the polynomial now, we have two terms remaining. Keep in mind that terms of a polynomial will always be separated by addition or subtraction. For this form, $2x(a+b)$ is the first term and $3(a+b)$ is the second term. We now go to Step 2: Factor out the GCF of the remaining two terms. If we look at these terms individually we can see that they both have a factor of $(a+b)$, this is our GCF. When we factor that out we get $(a+b)(2x+3)$ as the *2x* and the *+3* are what remains after we take out the GCF. We have now completely factored our four-term polynomial. Let's look at a couple more examples.

Example: Factor by grouping

a) $2xy+y^2-2x-y$ b) $xr+xs+yr+ys$ c) $x^3y^2-2x^2y^2+3xy^2-6y^2$

Solutions: Looking first at problem (a) we see that the first two terms have a *y* in common and the last two terms do not seem to have anything in common. If we look more closely, we can see that they both have a factor of *-1*; when nothing else seems common we can always factor out a 1 or -1. Rewriting after taking out these two GCFs we have $y(2x+y)-1(2x+y)$. Once again we are left with two terms and their GCF is already in parenthesis. We can finish by factoring into $(2x+y)(y-1)$.

(b) It seems as though this second one is fairly straight-forward, the first two terms share an *x* and the last two terms share a *y*. We can factor this to $x(r+s)+y(r+s)$ and finish it by taking out the GCF in parenthesis to give $(r+s)(x+y)$.

(c) The third example looks confusing, there is so much going on in each of the terms. Don't forget one of the most important tools we have, the GCF. Always look for an overall GCF before trying to factor by grouping. If we look at all four terms, we find that they all have a y^2 , therefore this is the overall GCF. Our first step is then $y^2(x^3-2x^2+3x-6)$. We now focus our attention on the portion that remains in the parenthesis. The first two have x^2 in common and the last two have 3 in common. Factoring these out gives $y^2(x^2(x-2)+3(x-2))$ as we have to keep our original GCF in front. Still looking inside the parenthesis, we find the GCF is $x-2$ and are able to finish factoring this as $y^2(x-2)(x^2+3)$.

Factoring Trinomials of the Form ax^2+bx+c - This method will work for every trinomial, whether the coefficient of the squared term is 1 or any other number; if the polynomial can be factored this will do it. The best way to explain the method is with the help of an example. Let's factor $2x^2+13x+15$. Looking at the general form we see that *a = 2, b = 13,* and *c = 15.* **Step 1**: Multiply *ac*. For this example (2)(15)=30. **Step 2**: Find two numbers, *m* and *n*, such that *mn=ac* and *m+n=b*. Now I need to find two numbers that multiply to be 30 and add to be 13. If necessary I could write down every factor pair (see Factoring Practice 1 for factoring numbers) of 30: 1,30; 2,15; 3,10; 5,6 and then choose the two that add to be 13 which are 3 and 10. **Step 3**: Rewrite the polynomial $ax^2+bx+c=ax^2+mx+nx+c$. For our example we have $2x^2+3x+10x+15$. **Step 4**: Use factoring by grouping to finish factoring. Our example has *x* as the GCF of the first pair and 5 as the GCF of the second pair to give $x(2x+3)+5(2x+3)$ which becomes $(2x+3)(x+5)$. The most challenging part of this method is step 2, you must be able to find factors of numbers. Let's try a few more.

Example: Factor completely.

(a) $t^2-11t+28$ (b) $7m^2-25m+12$ (c) $24x^2-44x-40$

Solutions: (a) In the first problem we multiply (1)(28) = 28. We need two numbers that multiply to be 28 and add to be -11. Because the 28 is positive and the 11 is negative, I know the two numbers must be negative. The factor pairs of 28 are 1,28; 2,14; 4,7. The last pair is the one I need so I rewrite the trinomial as $t^2-4t-7t+28$. (Note: the order of the factor pair will not matter.) I see that *t* is the GCF of the first two and 7 is the GCF of the last two. However, I want to use -7 as the GCF because the *7t* is negative; always use the middle sign. We now have $t(t-4)-7(t-4)=(t-4)(t-7)$.

(b) Multiply 7(12) = 84. Finding all factor pairs of 84 we get 1,84; 2,42; 3,28; 4,21 and we stop here. The 84 is positive so our two numbers must have the same sign. The *-25m* is negative so the two numbers must both be negative and add to -25. We have those numbers at -4 and -21 so we don't need to find more factor pairs. Rewrite and factor by grouping to get $7m^2-25m+12=7m^2-4m-21m+12=m(7m-4)-3(7m-4)=(7m-4)(m-3)$.

(c) If I were to multiply 24(-40) I think I would go crazy finding factor pairs. Once again I need to keep in mind the idea of always taking out the GCF of the overall polynomial. In this case each term has a 4 in common. Factor that out to get $4\left(6x^2-11x-10\right)$ which is much more manageable. Looking inside the parenthesis we multiply 6(-10)= - 60. We want to factors of -60 (so must have opposite signs) that add to be -11. This tells me that the two numbers must be different by 11 as one will be positive and one will be negative. The factor pairs of 60 are 1,60; 2,30; 3,20; 4,15 and we stop here. We have found two numbers, 4 and 15, that are different by 11. In order to multiply to be -60 one must be positive and one negative. How do we know which one is negative? Keep in mind they must add to be -11 so it must be +4 and -15. Rewrite and factor, keeping the original GCF out front:

$$\begin{aligned} 4\left(6x^2-11x-10\right) &= 4\left(6x^2+4x-15x-10\right) \\ &= 4\left(2x(3x+2)-5(3x+2)\right) \\ &= 4(3x+2)(2x-5) \end{aligned}$$

This method will work for every factorable polynomial every single time. It takes practice but with practice you can learn to do it. (You may never be as fast as some of your classmates, but speed isn't all that important.)

Practice Problems – Completely factor each polynomial.

1. $x^2+11x+10$	2. $x^2-10x+21$
3. x^2-6x+8	4. $x^2+12x+36$
5. $15+2x-x^2$	6. $14+5x-x^2$
7. $3x^2-12x-36$	8. x^3+8x^2-20x
9. $y^4+11y^3+30y^2$	10. $3y^3-18y^2-48y$
11. $6x^2+8x+2$	12. $8x^2+6x-2$
13. $10x^2-11x-6$	14. $6x^2+23x+20$
15. $6x^2+11x-7$	16. $5x^2+7x-6$

Practice Solutions – Keep in mind the order of the factors does not matter.

1. $(x+10)(x+1)$
2. $(x-7)(x-3)$
3. $(x-4)(x-2)$
4. $(x+9)(x+4)$
5. $(5-x)(3+x)$
6. $(7-x)(2+x)$
7. $3(x-6)(x+2)$
8. $x(x+10)(x-2)$
9. $y^2(y+6)(y+5)$
10. $3y(y-8)(y+2)$
11. $2(3x+1)(x+1)$
12. $2(4x-1)(x+1)$
13. $(5x+2)(2x-3)$
14. $(3x+4)(2x+5)$
15. $(2x-1)(3x+7)$
16. $(x+2)(5x-3)$