1.3 Linear Functions and Models

Definition – A linear function is one that can be written in the form $f(x) = mx + b$ or $y = mx + b$ where *m* and *b* are fixed numbers.

Role of *m*: If $y = mx + b$, then *y* changes by *m* units for every 1 unit change in *x*. A change of Δx units in *x* results in a change of $\Delta y = m\Delta x$ units in *y*. Thus

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$$

Role of *b*: Numerically, when $x = 0, y = b$. This naturally leads to the graphical role which is that $(0, b)$ is the *y*-intercept of the graph of $y = mx + b$.

Example: Decide which of the given functions are linear. Use your knowledge of slope and y-intercept to write the equation of each linear function.

x	-2	-1	0	1	2	3	4
f(x)	1	4	7	10	13	16	19
g(x)	8	3	-2	-7	-12	-17	-22
h(x)		6		10		14	
j(x)	9		4		0		-3

This will be discussed in detail in class.

Examples: Find the slope, if defined.

1. $y = \frac{2x}{3} + 4$

$\frac{2x}{3} = \frac{2}{3}x$ so $m = \frac{2}{3}$

2. $8x - 2y = 1$

$\frac{-2y}{-2} = \frac{-8x}{-2} + \frac{1}{-2}$

$y = 4x - \frac{1}{2}$ $\quad m = 4$

3. $2y + 3 = 0$

$2y = -3$

$y = -\frac{3}{2}$

or $y = 0x - \frac{3}{2}$ $\quad m = 0$

4. $3x + 5 = 0$

we cannot solve for y

so m is undefined

5. You try it: $2x - 4y = 7$

Examples: Calculate the slope, if defined.

1. $(0,0)$ *and* $(-1,2)$

$m = \frac{2-0}{-1-0} = \frac{2}{-1} = -2$

2. $(4,3)$ *and* $(4,1)$

$m = \frac{1-3}{4-4} = \frac{-2}{0}$ uh oh!

m is undefined

0 in denominator is bad

0 in numerator is ok

3. $(-2,4)$ *and* $(3,7)$

$m = \frac{7-4}{3-(-2)} = \frac{7-4}{3+2} = \frac{3}{5}$

4. $(4,3)$ *and* $(1,3)$

$m = \frac{3-3}{1-4} = \frac{0}{-3} = 0$

5. You try it: $(-1,8)$ *and* $(5,17)$

Examples: Find a linear equation whose graph is the straight line with the given properties.

1. Through $(2,1)$ with slope 2

$m=2$ $\quad x=2, y=1$

Start with $y=mx+b$

$1 = 2(2)+b$

$1 = 4+b$

$-3 = b$

$y = 2x-3$

2. Through $\left(0,-\frac{1}{3}\right)$ with slope $\frac{1}{4}$

$m = \frac{1}{4}$

$y = mx+b$

$-\frac{1}{3} = \frac{1}{4}(0)+b$

$-\frac{1}{3} = b$

$y = \frac{1}{4}x - \frac{1}{3}$

3. Through $(2,-4)$ *and* $(1,1)$

$m = \frac{1-(-4)}{1-2} = \frac{1+4}{1-2} = \frac{5}{-1} = -5$

$y = mx+b$

$1 = -5(1)+b$

$1 = -5+b$

$6 = b$

$y = -5x+6$

Try it with $(2,-4)$.

4. You try it: Through $(1,-4)\ and\ (2,5)$

Example: The Ride-Em Bicycles factory can produce 100 bicycles in a day at a total cost of $11,400 and it can produce 140 bicycles in a day at a total cost of $12,200. What are the company's daily fixed costs, and what is the marginal cost per bicycle?

find m   slope is $\frac{\text{cost}}{\text{bike}} \rightarrow$ $y=\text{cost}$, $x=\#\text{bicycles}$

$(100,\ 11400)$
$(140,\ 12200)$ find b

$$m=\frac{12200-11400}{140-100}=\frac{800}{40}=\$20/\text{bicycle}$$

$$11400=20(100)+b$$
$$11400=2000+b$$
$$9400=b \rightarrow$$ daily fixed costs are $9,400.

Example: You can sell 60 pet chias per week if they are marked at $1 each, but only 20 each week if they are marked at $2/chia. Your chia supplier is prepared to sell you 15 chias per week if they are marked at $1/chia, and 95 each week if they are marked at $2/chia.

demand

supply

a) Write down the associated linear demand and supply functions in the form $q = mp + b$.

input = p   output = q   so points are $(1,60)$ and $(2,20)$

$$m=\frac{20-60}{2-1}=\frac{-40}{1}=-40$$

$$60=-40(1)+b$$
$$60=-40+b$$
$$100=b$$
$\rightarrow$ demand is $q=-40p+100$

$(1,15)$ and $(2,95)$

$$m=\frac{95-15}{2-1}=\frac{80}{1}=80$$

$$15=80(1)+b$$
$$15=80+b$$
$$-65=b$$
$\rightarrow$ supply is $q=80p-65$

b) At what price should the chias be marked so that there is neither a surplus nor a shortage of chias?

$$-40p+100=80p-65$$
$$+40p \quad +65 \quad +40p \quad +65$$
$$165=120p$$

$$\frac{165}{120}=p$$
$$p=1.375$$

Sell pet Chias for $1.38.

Linear Change over Time – If a quantity *q* is a linear function of time t, $q = mt + b$, then the slope *m* measures the rate of change of *q*, and *b* is the quantity at time $t=0$, the initial quantity. If *q* represents the position of a moving object, then the rate of change is also called the velocity.