2.2 Exponential Functions and Models

**The Laws of Exponents** – If *b* and *c* are positive and *x* and *y* are any real numbers, then the following laws hold:

1. $b^x b^y = b^{x+y}$ When you multiply two things with the same base you add their exponents.

2. $\dfrac{b^x}{b^y} = b^{x-y}$ When you divide two things with the same base you subtract their exponents.

3. $\dfrac{1}{b^x} = b^{-x}$ and $\dfrac{1}{b^{-x}} = b^x$

4. $b^0 = 1$

5. $\left(b^x\right)^y = b^{xy}$ When you raise a power to a power, you multiply.

6. $(bc)^x = b^x c^x$

7. $\left(\dfrac{b}{c}\right)^x = \dfrac{b^x}{c^x}$

**Definition** – An exponential function has the form $f(x) = A(b)^x$ where *A* and *b* are constants with $A \neq 0$ and $b > 0$ with $b \neq 1$.

Role of *A*: $f(0) = A$ , so *A* is the *y*-intercept of the graph of *f*

Role of *b*: If *x* increases by 1, $f(x)$ is multiplied by *b*.

Examples: Create a table of values.

1. $f(x) = 3(2)^x$

| x | y | |
|---|---|---|
| $-2$ | $3(2)^{-2} = 3(\frac{1}{4}) = \frac{3}{4}$ | $\times 2$ |
| $-1$ | $3(2)^{-1} = 3(\frac{1}{2}) = \frac{3}{2}$ | $\times 2$ |
| $0$ | $3(2)^0 = 3(1) = 3$ | $\times 2$ |
| $1$ | $3(2)^1 = 3(2) = 6$ | $\times 2$ |
| $2$ | $3(2)^2 = 3(4) = 12$ | |

base = 2, next y is x2

2. $g(x) = -2\left(\dfrac{1}{3}\right)^x$

| x | y | |
|---|---|---|
| $-2$ | $-2(\frac{1}{3})^{-2} = -2(9) = -18$ | $\times \frac{1}{3}$ |
| $-1$ | $-2(\frac{1}{3})^{-1} = -2(3) = -6$ | $\times \frac{1}{3}$ |
| $0$ | $-2(\frac{1}{3})^0 = -2(1) = -2$ | $\times \frac{1}{3}$ |
| $1$ | $-2(\frac{1}{3})^1 = -2(\frac{1}{3}) = -\frac{2}{3}$ | $\times \frac{1}{3}$ |
| $2$ | $-2(\frac{1}{3})^2 = -2(\frac{1}{9}) = -\frac{2}{9}$ | |

base = $\frac{1}{3}$, next y is $\times \frac{1}{3}$

Example: The values of several functions are given in a table. Decide which are exponential and then find their equation.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 0.5 | 1.5 | 4.5 | 13.5 | 40.5 |
| $g(x)$ | 8 | 4 | 2 | 1 | ½ |
| $h(x)$ | 100 | 200 | 400 | 600 | 800 |
| $j(x)$ | 0.3 | 0.9 | 2.7 | 8.1 | 24.3 |

This will be discussed in class.

Examples: Find equations for exponential functions of the form $y = A(b)^x$ that pass through the given points. Round all coefficients to 4 decimal places, if necessary.

1. $(2,36)$ and $(4,324)$

$36 = A(b)^2$ | $324 = A(b)^4$

$\frac{36}{b^2} = A$ → $324 = \frac{36}{b^2}(b^4)$

$324 = \frac{36b^4}{b^2}$

$\frac{324}{36} = \frac{36b^2}{36}$

$9 = b^2$

$\pm 3 = b$

b must be positive $b=3$

$\frac{36}{3^2} = A$

$\frac{36}{9} = A$

$4 = A$

$\boxed{y = 4(3)^x}$

2. $(2,-4)$ and $(4,-16)$

$-4 = A(b)^2$ | $-16 = A(b)^4$

① $\frac{-4}{b^2} = A$ → ② $-16 = \frac{-4}{b^2}(b^4)$

$-16 = \frac{-4b^4}{b^2}$

$-16 = -4b^2$

$4 = b^2$

$2 = b$

③ $\frac{-4}{4} = A$

$-1 = A$

$\boxed{y = -1(2)^x}$

3. You try it: $(1,3)$ and $(3,6)$

**Definition** – The number *e* is the limiting value of the quantities $\left(1+\frac{1}{m}\right)^m$ as *m* gets larger and larger and has the value of $e \approx 2.71828182845904523536...$ If \$*P* is invested at an annual interest rate *r* compounded continuously, the accumulated amount after *t* years is $A(t) = Pe^{rt}$.

Example: Rock Solid Bank & Trust is offering a CD that pays 4% compounded continuously. How much interest would a \$1,000 deposit earn over 10 years?

$P = 1000, \quad r = 4\% = 0.04, \quad t = 10$

$A = 1000e^{(0.04 \cdot 10)} = 1491.82$

$I = A - P = 1491.82 - 1000 = \boxed{\$491.82}$