

Chapter Three: The Mathematics of Finance

3.1 Simple Interest

Definition – The simple interest on an investment (or loan) of PV dollars at an annual interest rate of r for a period of t years is $INT = PVrt$.

Definition – The future value of an investment of PV dollars at an annual simple interest rate of r for a period of t years is given by $FV = PV + INT$ which can be simplified to $FV = PV(1 + rt)$.

Example: Compute the simple interest and find the future value.

1. \$2000 is invested for 10 years at 4% per year.

$$INT = 2000(.04)(10) = \$800$$

$$FV = 2000 + 800 = \$2800$$

2. \$1000 is invested for 6 months at 5% per year.

$$INT = 1000(.05)\left(\frac{6}{12}\right) = \$25$$

$$FV = 1000 + 25 = \$1025$$

we use $\frac{6}{12}$ for t because 6 months is $\frac{6}{12}$ of a year

3. You try it: \$10,000 is invested for 3 months at 11% per year.

Example: Find the present value.

1. An investment earns 2% per year and is worth \$10,000 after 5 years.

$$10,000 = PV(1 + .02(5))$$
$$\frac{10,000}{(1 + .02(5))} = PV \quad \rightarrow \quad PV = \frac{10,000}{1.1} = \$9090.91$$

2. An investment earns 7% per year and is worth \$1000 after 6 months.

$$1000 = PV\left(1 + .07\left(\frac{6}{12}\right)\right)$$
$$1000 = PV(1 + .035)$$
$$\frac{1000}{1.035} = PV \quad \rightarrow \quad PV = \$966.18$$

3. You try it: An investment earns 6% per year and is worth \$30,000 after 20 months.

Example: The simple interest on a $\$1000$ loan at 8% per year amounted to $\$640$. When did the loan mature?

$$\overbrace{PV} \quad \overbrace{r=0.08} \quad \overbrace{INT} \quad \xrightarrow{\text{find } t}$$

$$INT = PVr t \text{ becomes } 640 = 1000(.08)t$$

$$640 = 80t$$

$$\frac{640}{80} = t$$

It matured in 8 years.

Example: You take out a 2-year, $\$5000$ loan at 9% simple annual interest. The lender charges you a $\$100$ fee. Thinking of the fee as additional interest, what is the actual annual interest rate you will pay?

$$\text{First sentence: } t=2, PV=5000, r=9\%=0.09$$

$$\$100 \text{ fee is additional interest so } INT = 5000(.09)(2) = 900$$

$$\text{Total fee/interest is } 900 + 100 = 1000.$$

$$INT = 1000, t=2, PV=5000, \text{ find } r$$

$$1000 = 5000r(2)$$

$$1000 = 10,000r$$

$$\frac{1000}{10,000} = r$$

$$\longrightarrow r = \frac{1}{10} = 0.10 = 10\%$$