

4.3 Applications of Systems of Linear Equations

Example 1 – You manage an ice cream factory that makes three flavors: Creamy Vanilla (CV), Continental Mocha (CM), and Succulent Strawberry (SS). Into each batch of CV go 2 eggs, 1 cup of milk, and 2 cups of cream. Into each batch of CM go 1 egg, 1 cup of milk, and 2 cups of cream, while into each batch of SS go 1 egg, 2 cups of milk, and 1 cup of cream. You have 350 eggs, 350 cups of milk, and 400 cups of cream. How many batches of each flavor should you make in order to use up all your ingredients?

Last sentence, the question/action tells our variables:

How many batches of each Flavor. Let x = batches of CV,
 y = batches of CM, z = batches of SS.

Totals tell equations: 350 eggs, 350 milk, 400 cream

eggs: $2x + y + z = 350$

milk: $x + y + 2z = 350$

cream: $2x + 2y + z = 400$

①+②

$$2x + y + z = 350$$

$$-2x - 2y - 4z = -700$$

$$\hline -y - 3z = -350$$

①+③

$$-2x - y - z = -350$$

$$2x + 2y + z = 400$$

$$\hline y = 50$$

Now that we know $y = 50$, we can find z :

$$-50 - 3z = -350$$

$$\hline +50 \qquad +50$$

$$-3z = -300$$

$$z = 100$$

We know $y = 50$, $z = 100$ so we can

find x :

$$x + y + 2z = 350$$

$$x + 50 + 2(100) = 350$$

$$x + 250 = 350$$

$$x = 100$$

Answer:

100 batches CV

50 batches CM

100 batches SS

Example 2 – You own a hamburger franchise and are planning to shut down operations for the day, but you are left with 13 buns, 19 defrosted beef patties, and 15 open cheese slices. Rather than throw them out, you decide to use them to make burgers that you will sell at a discount. Plain burgers each require 1 beef patty and 1 bun, double cheeseburgers each require 2 beef patties, 1 bun, and 2 slices of cheese, while regular cheeseburgers each require 1 beef patty, 1 bun, and 1 slice of cheese. How many of each type of burger should you make?

$x = \text{number of plain}$, $y = \text{number of Cheese}$, $z = \text{number dbl meat dbl cheese}$

Equations:

buns $x + y + z = 13$

beef $x + y + 2z = 19$

cheese $y + 2z = 15$

$$\begin{array}{r} \textcircled{1} - \textcircled{2} \\ -x - y - z = -13 \\ \underline{x + y + 2z = 19} \\ z = 6 \end{array}$$

With $z=6$ we have $y + 2(6) = 15$, so $y=3$.

The top equation allows us to find x : $x + 3 + 6 = 13$
 $x = 4$

We should sell 4 plain burgers, 3 cheese burgers, and 6 double cheeseburgers.

Example 3 – The Fancy French Perfume Company recently had its secret formula divulged. It turned out that it was using, as the three ingredients, rose oil, oil of fermented prunes, and alcohol. Moreover, each 22-ounce econo-size bottle contained 4 more ounces of alcohol than oil of fermented prunes, while the amount of alcohol was equal to the combined volume of the other two ingredients. How much of each ingredient did it use in the econo-size bottle?

$x = \text{amount of rose oil}$
 $y = \text{amount of prunes}$
 $z = \text{amount of alcohol}$

} in ounces

$$x + y + z = 22$$

$$-y + z = 4$$

$$-x - y + z = 0$$

Solve: $\textcircled{1} + \textcircled{2}$

$$x + y + z = 22$$

$$-x - y + z = 0$$

$$\hline 2z = 22$$

$$z = 11$$

\rightarrow If $z = 11$, $-y + 11 = 4$ so $y = 7$.

Knowing y and z we have $x + 7 + 11 = 22$ so $x = 4$.

We use 4 ounces of rose oil, 7 ounces of fermented prune oil, and 11 ounces of alcohol.

more alcohol, so to make them equal $z = y + 4$

Rewrite: $-y + z = 4$

alcohol = prunes + rose oil

$$z = y + x$$

Rewrite: $-x - y + z = 0$

Example 4 – You invested a total of \$8400 in shares of three stocks at the prices listed, and expected to earn \$248 in annual dividends. If you purchased a total of 200 shares, how many shares of each stock did you purchase?

	Stock	Price per share	Dividend Yield
X	GE	\$16/share	7%
Y	WMT	\$56/share	2%
Z	XOM	\$80/share	2%

$$\text{Shares: } x + y + z = 200$$

$$\text{Invest: } 16x + 56y + 80z = 8400$$

$$\text{Dividend: } .07x + .02y + .02z = 248$$

you finish it!