7.3 Decision Algorithms: The Addition and Multiplication Principles

Definition: When choosing among *r* disjoint alternatives, if alternative 1 has n_1 outcomes, alternative 2 has n_2 outcomes, …, alternative *r* has n_r different outcomes, then you have a total of $n_1 + n_2 + \ldots + n_r$ possible outcomes.

Example: A restaurant offers 3 main course salads, 4 pasta dishes, 3 beef meals, 2 chicken dinners, and 1 pork meal. This gives a total of 3 + 4 + 3 + 2 + 1 = 13 main courses to choose from.

Definition: When making a sequence of choices with *r* steps, if step 1 has n_1 possible outcomes, step 2 has n_2 outcomes, …, step *r* has n_r possible outcomes, then you have a total of $n_1 \times n_2 \times \ldots \times n_r$ possible outcomes.

Example: Hells Kitchen offers 3 appetizers, 5 main courses, and 2 desserts. You can create 3 x 5 x 2 = 30 different three course meals with that menu.

Example: A surgical procedure requires choosing among four alternative methodologies. The first can result in four possible outcomes, the second in three possible outcomes, and the remaining methodologies can each result in two possible outcomes. What is the total number of outcomes possible?

alternatives = options = add

$4+3+2+2 = 11$ outcomes

Example: A surgical procedure requires four steps. The first can result in four possible outcomes, the second can result in three possible outcomes, and the remaining two can each result in two possible outcomes. What is the total number of outcomes possible?

steps rely on a sequence = multiply

$4 \times 3 \times 2 \times 2 = 48$ outcomes

Definition: A decision algorithm is a procedure in which we make a sequence of decisions. We can use decision algorithms to determine the number of possible items by pretending we are designing such an item and listing the decisions or choices we should make at each stage of the process.

Example: Find how many outcomes are possible:

Alternative 1: Step 1 has 1 outcome, step 2 has 2 outcomes

Alternative 2: Step 1 has 2 outcomes, step 2 has 2 outcomes, and step 3 has 1 outcome.

add alternatives $\quad$ Alt 1 + Alt 2

multiply steps: $\quad 1 \times 2 + 2 \times 2 \times 1$

$2 + 4 = 6$ total outcomes

Example: Professor Easy's final exam has 10 true-false questions followed by 2 multiple-choice questions. In each of the multiple choice questions, you must select the correct answer from a list of five. How many answer sheets are possible?

followed by = sequence = multiply

T/F has 2 options, 10 questions $= 2^{10}$

MC has 5 options, 2 questions $= 5^2$

$2^{10} \times 5^2$

$= 1024 \times 25$

$= 25{,}600$ answer sheets

Example: A test requires that you answer either Part A or Part B. Part A consists of eight true-false questions, and Part B consists of five multiple choice questions with one correct answer in four. How many different completed answer sheets are possible?

or = alternatives = add

Part A $\quad$ 2 options, 8 questions $= 2^8$

Part B $\quad$ 4 options, 5 questions $= 4^5$

$2^8 + 4^5 = 256 + 1024 = 1280$