8.4 Probability and Counting Techniques

Examples: Suzy sees a bag containing 4 red marbles, 3 green marbles, 2 white marbles, and 1 purple marble. She grabs 5 of them. Find the probabilities of the following events, expressing each as a fraction in lowest terms.

10 marbles, grab 5 $\quad n(S) = C(10,5) = 252$

1. She has all the red ones.

$C(4,4)\cdot C(6,1) = 1\cdot 6 = 6 \qquad P(\text{all red}) = \frac{6}{252} = \frac{1}{42}$

2. She has none of the red ones.

$C(4,0)\cdot C(6,5) = 1\cdot 6 = 6 \qquad P(\text{no red}) = \frac{6}{252} = \frac{1}{42}$

3. She has at least 1 white one.

at least 1 is 1 or 2

$C(2,1)\cdot C(8,4) + C(2,2)\cdot C(8,3)$

$2\cdot 70 + 1\cdot 56 = 140 + 56 = 196$

$P(\text{at least 1 white}) = \frac{196}{252} = \frac{7}{9}$

4. She has 2 reds and 1 of each other color.

$C(4,2)\cdot C(3,1)\cdot C(2,1)\cdot C(1,1) = 6\cdot 3\cdot 2\cdot 1 = 36$

$P(\text{this}) = \frac{36}{252} = \frac{1}{7}$

5. She has at most 1 green one.

0 or 1 green

$C(3,0)\cdot C(7,5) + C(3,1)\cdot C(7,4)$

$= 1\cdot 21 + 3\cdot 35 = 21 + 105 = 126$

$P(\text{at most 1 green}) = \frac{126}{252} = \frac{1}{2}$

Examples: Poker is a game that consists of dealing 5 cards at random from a standard deck of 52. Find the probability of each hand. $n(S) = C(52,5) = 2{,}598{,}960$

13 denominations (values)

4 suits

1. Two of a kind: 2 cards with the same denomination and 3 cards with other denominations (different from each other and different from the pair).

$C(13,1)\cdot C(4,2)\cdot C(12,3)\cdot C(4,1)C(4,1)C(4,1)$

$= 13\cdot 6\cdot 220\cdot 4\cdot 4\cdot 4 = 1098240$

$\frac{1098240}{2598960} \approx 0.4226$

2. Three of a kind: 3 cards of the same denomination and 2 cards with other denominations.

$C(13,1)\cdot C(4,3)\cdot C(12,2)\cdot C(4,1)C(4,1)$

$= 13\cdot 4\cdot 66\cdot 4\cdot 4 = 54912$

$P(3\text{ of kind}) = \frac{54192}{2598960} \approx 0.0211$

3. Two pair: 2 cards with one denomination, 2 cards with another, and 1 with a third.

$C(13,1)\cdot C(4,2)\cdot C(12,1)\cdot C(4,2)\cdot C(11,1)\cdot C(4,1)$

$= 13\cdot 6\cdot 12\cdot 6\cdot 11\cdot 4 = 247{,}104$

$P(2\text{ pair}) = \frac{247{,}104}{2598960} = 0.0951$

None of these make it worth wagering big money... unless you have it to lose.

Example: A test has three parts. Part A consists of eight true-false questions, Part B consists of five multiple choice questions with five choices each, and Part C requires you to match five questions with five different answers one-to-one. Assuming that you make random guesses in filling out your answer sheet, what is the probability that you will earn 100% on the test?

you must do all three parts

$$2^8 \cdot 5^5 \cdot 5! = 256 \cdot 3125 \cdot 120 = 96{,}000{,}000$$

$$P(100\%) = \frac{1}{96 \text{ million}}$$

(study to improve odds)

Example: The Random Example Lottery requires you to select a sequence of three different numbers from 0 through 49. (Order is important.) You are a Winner if your sequence agrees with that in the drawing, and you are a Booby Prize Winner if your selection of number is correct, but in the wrong order. What is the probability of being a Winner? What is the probability of being a Booby Prize Winner? What is the probability that you are either a Winner or a Booby Prize Winner?

Winner: $P(50,3) = 117{,}600$

$$P(\text{winner}) = \frac{1}{117{,}600} \approx 0.000\,008\,503$$

Booby prize: $C(50,3) = 19{,}600$

$$P(\text{B. prize}) = \frac{1}{19{,}600} \approx 0.000\,051\,020$$

Neither winner nor Booby prize winner is

$$1 - P(\text{win}) - P(\text{B. prize}) = 1 - \frac{1}{117{,}600} - \frac{1}{19{,}600} \approx 0.999\,940$$