8.5 Conditional Probability and Independence

Definition: If *A* and *B* are events with $P(B) \neq 0$, then the probability of *A* given *B* is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Examples: Compute the indicated quantity.

1. $P(B) = 0.5, P(A \cap B) = 0.2, P(A|B) = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = 0.4$$

2. $P(A|B) = 0.2, P(B) = 0.4, P(A \cap B) = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ so } 0.2 = \frac{P(A \cap B)}{0.4} \rightarrow 0.2(0.4) = P(A \cap B)$$

$$0.08 = P(A \cap B)$$

3. $P(A|B) = 0.4, P(A \cap B) = 0.3, P(B) = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ so } 0.4 = \frac{0.3}{P(B)}$$

$$0.4P(B) = 0.3$$

$$P(B) = \frac{0.3}{0.4} = 0.75$$

Examples: Find the conditional probabilities of the indicated events when two fair distinguishable dice are rolled. (See handout dice coins cards for full dice chart.)

1. The sum is 5, given that the green one is not 1.

$$P(\text{sum } 5 \mid \text{gr} \neq 1) = \frac{P(\text{sum } 5 \cap \text{green} \neq 1)}{P(\text{gr} \neq 1)} = \frac{3/36}{30/36} = \frac{3}{30} = \frac{1}{10}$$

(1 4)
(2 3)
(3 2)
~~(4 1)~~

2. The red one is 5, given that the sum is 6.

$$P(5 \mid \text{sum } 6) = \frac{P(5 \cap \text{sum } 6)}{P(\text{sum } 6)} = \frac{1/36}{5/36} = \frac{1}{5}$$

Sum is 6: 1 5, 2 4, 3 3, 4 2, 5 1

3. The sum is 6, given that the dice have opposite parity. one even, one odd

Sum is 6 listed above

$$P(\text{sum } 6 \mid \text{opp parity}) = \frac{P(\text{sum } 6 \cap \text{opp parity})}{P(\text{opp parity})} = \frac{0/36}{18/36} = 0$$

Definition: If *A* and *B* are events, then $P(A \cap B) = P(A|B)P(B)$.

Definition: The events *A* and *B* are independent if $P(A \cap B) = P(A)P(B)$. If two events are not independent, they are said to be dependent.

Examples: The table shows the results of a survey of 100 authors by a publishing company.

	New Authors	**Established Authors**	**Total**
Successful	5	25	30
Unsuccessful	15	55	70
Total	20	80	100

Compute the following conditional probabilities.

1. An author is established, given that she is successful.

$$\frac{P(E \cap S)}{P(S)} = \frac{25}{30} = \frac{5}{6}$$

2. An author is successful, given that she is established.

$$\frac{P(S \cap E)}{P(E)} = \frac{25}{80} = \frac{5}{16}$$

3. An author is a new author, given that he is unsuccessful.

$$\frac{P(N \cap U)}{P(U)} = \frac{15}{70} = \frac{3}{14}$$

4. An unsuccessful author is established.

Rewrite as: the probability that an author is established given that they are unsuccessful.

$$\frac{P(E \cap U)}{P(U)} = \frac{55}{70} = \frac{11}{14}$$

5. An established author is successful.

$$\frac{P(S \cap E)}{P(E)} = \frac{25}{80} = \frac{5}{16}$$